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Optics, resonators and beams

D. Schüöcker and K. Schröder

2.1 THE KIRCHHOFF FRESNEL INTEGRAL

If a certain area $A$ of arbitrary curvature and shape is illuminated with linearly polarised light with electric field strength $E$, all points emit spherical waves according to Huygen's law. The electric field strength generated at a point $P$ distant from the illuminated area results from the superposition of all these spherical waves. If an infinitely small area $dA$ is considered, and the electric field strength in this area is approximately constant and equal to $E$, the contribution to the field at $P$ is proportional to $E.dA$, since the electric field strength determines the light emitted per unit area. This contribution reduces with increasing distance $\rho$ from the emitting surface element $dA$, due to conservation of energy, as the wave energy distributed across the spherical wave front remains constant, while the radius increases during propagation. The field strength at point $P$ is reduced by a factor $1/\rho$ since the light intensity is given by the square of the electric field strength. There is also a phase difference between the waves arriving at $P$, and those at the origin $dA$, given by $ik\rho$, due to the propagation time between $dA$ and $P$ according to equation (2.1). Finally, the electric field strength generated at $P$ due to the emission of the area element $dA$ also depends on the angle $\phi$ between the beam from $dA$ to $P$, and the area element $dA$. The maximum contribution is obtained when the beam is perpendicular to the area element. An angle $\phi$ greater than zero between the vector normal to $dA$ and the direction between $dA$ and $P$, reduces the area which is seen from the point $P$. 

D. Schüöcker (ed.), Handbook of the Eurolaser Academy
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This reduction follows the well known cosine law, which together with the assumptions made above yields:

\[ dE(P) = \text{const} \frac{1}{\rho} E(A) \exp(-ik\rho) \cos\varphi dA \]  \hspace{1cm} (2.1)

i.e. the infinitesimal contribution of the illuminated area dA to the wave at point P. A more sophisticated wave theoretical treatment leads to the term \(-i/2\lambda \ (1+\cos\varphi)\) which replaces the term \(\text{const} \times \cos\varphi\) in equation (2.1). Finally all the contributions of the individual elements must be summed to obtain the contribution of the whole illuminated area. As a result the following integral can be derived:

\[ E(P) = -\frac{i}{\lambda} \int_A \frac{1}{\rho} E(A) \exp(-ik\rho) \frac{(1+\cos\varphi)}{2} dA \]  \hspace{1cm} (2.2)

This integral is called the **Kirchhoff Fresnel** integral for the analytical description of diffraction. It is of crucial importance for the mathematical analysis of optical elements and their application in optical resonators.

### 2.2 FOURIER TRANSFORMATION BY FOCUSING

According to geometrical optics, application of the reflection law yields equal angles between an incident beam and the reflecting surface, and the reflecting surface and the reflected beam. This shows that a concave spherical mirror focuses light beams that are initially parallel to the axis of symmetry to a focus halfway between the mirror and its centre of curvature. The focal length of the mirror is thus one half of the radius of curvature R.

\[ f = \frac{R}{2} \]  \hspace{1cm} (2.3)

So far, geometrical optics yields a point like focus, which means that the intensity becomes infinitely large, an unrealistic result, since diffraction, leading to a slight bending of the beams of geometrical optics, has not been taken into account. Therefore the use of the Kirchhoff Fresnel integral that has been derived above is necessary.