GUIDED TRANSIENT WAVES IN A CIRCULAR ANNULUS

Guoli Liu and Jianmin Qu

The George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0405

INTRODUCTION

Fatigue cracks have been found to initiate and grow in the radial direction in many annulus shaped components in aging helicopters. Detection of such radial cracks using conventional ultrasonic imaging techniques is rather ineffective. Recently, it has been proposed [1] that guided ultrasonic waves that propagate in the circumferential direction may be used for the detection of radial fatigue cracks in weep holes in airframes. Before such guided waves can be used effectively for detecting radial cracks in annulus components, their generation and propagation must be understood. The physics and mechanics of the wave fields must be carefully examined.

A comprehensive review of studies on wave propagation in the axial direction of a cylinder can be found in [2]. A more recent work by Ditri and Rose [3] solved the problem of transient wave propagation in the axial direction of a hollow cylinder subjected to surface traction. The dispersion of elastic waves and Rayleigh-type waves in a thin disc was investigated by Červ [4]. Recently, Liu and Qu derived the dispersion equation for the time-harmonic circumferential waves in an annulus [5]. The displacement profiles across the thickness were analyzed for various modes. It was found that the first and second modes correspond to the surface Rayleigh wave propagating along outer and inner surfaces, respectively [6]. The other higher modes are truly propagating guided waves formed by the multiple reflection from both surfaces.

In this paper, transient circumferential waves in a 2-D circular annulus are considered. The outer surface of the annulus is subjected to a time-dependent impulse excitation. The guided circumferential waves induced by this impulse are investigated by using the method of eigenfunction expansion. The steady-state, time harmonic circumferential waves are solved first as the eigenfunctions. The time-dependent response of the annulus is then obtained by superimposing all eigenfunctions over all possible frequencies. Several numerical examples are given to illustrate the method of solution.
PROBLEM STATEMENT

Consider an annulus of inner radius $a$ and outer radius $b$ as shown in Fig. 1. It is assumed that the material is linearly elastic and isotropic. For the two dimensional deformation studied here, assume that plane strain deformation in the annulus prevails. Therefore, in the polar coordinate system $(r, \theta)$ shown in Fig. 1, the displacement components in the annulus can be written as

$$\mathbf{u} = \begin{bmatrix} u_r \\ u_\theta \end{bmatrix} = \begin{bmatrix} u_r(r, \theta, t) \\ u_\theta(r, \theta, t) \end{bmatrix}.$$ (1)

Then, the displacement equation of motion without body force is [7]

$$L[\mathbf{u}] = c_L^2 \nabla (\nabla \cdot \mathbf{u}) - c_T^2 \nabla \times \nabla \times \mathbf{u} = \frac{\partial^2 \mathbf{u}}{\partial t^2}, \text{ in } \mathcal{V}$$ (2)

where $\mathcal{V}$ denotes the annulus, $c_L$ and $c_T$ are the longitudinal and shear wave phase velocities, respectively.

On the boundary of $\mathcal{V}$, $\mathbf{u}$ must satisfy the following boundary conditions

$$B[\mathbf{u}] = \lambda (\nabla \cdot \mathbf{u}) \cdot \mathbf{n} + \mu [\nabla \mathbf{u} + (\nabla \mathbf{u})^T] \cdot \mathbf{n} = \mathbf{F} \text{ on } S,$$ (3)

Fig. 1 A circular annulus with inner radius $a$, outer radius $b$, subjected to a time-dependent force on the outer surface.