INTRODUCTION

Lamb wave ultrasonic testing has been employed as a practical nondestructive method to detect defects in a thin plate. The quantitative evaluation of the ultrasonic testing has not, however, been established since the scattering process of elastic waves in a plate depends on various testing conditions such as frequencies, wave modes, a plate thickness, defect’s properties and so on. Therefore, numerical simulations for the wave scattering in a plate are necessary to make the ultrasonic method more quantitative.

Lamb wave scattering problems by edges or defects in a plate have been investigated using various methods such as the T-matrix method [1], the boundary element method (BEM) combined with the normal mode technique [2,3] and the hybrid method of the finite element method and the Lamb wave modal expansions [4]. Hirose and Yamano [5] developed the BEM to solve the scattering problems of Lamb waves by a crack and investigated the backscattered far-fields mode by mode. In their BEM formulation, the fundamental solution for a full space was used even for a plate problem. Since the fundamental solution does not satisfy the boundary conditions on the upper and bottom surfaces of a plate, the BEM requires the integrations over the plate surfaces as well as the integrations over the boundaries of defects. Therefore the system of equations to be solved becomes large and the truncation error is induced in the solutions.

In this paper, a boundary element method (BEM) with the Green’s function is presented for an SH wave scattering by a crack in a plate. The Green’s function satisfies the traction free conditions on plate surfaces. Hence, the boundary integral equations are discretized only over the boundaries of defects.

PROBLEM STATEMENT AND BEM FORMULATION

Let $D$ be a two dimensional domain of a homogeneous, isotropic, linearly elastic plate with the thickness $2h$ including a crack $S$ as shown Fig. 1. The wave field is
assumed to be in an antiplane state with the time factor $e^{-i\omega t}$, where $\omega$ is the angular frequency. In the following, the time factor $e^{-i\omega t}$ will be omitted.

The governing equation for the antiplane displacement $u$ is given by

$$\nabla^2 u(\vec{x}) + k_T^2 u(\vec{x}) = 0, \quad \vec{x} \in D$$

where $k_T$ is the wavenumber of the transverse wave. The traction free boundary conditions are given on the crack face $S$ and on the upper and bottom surfaces $B$ of the plate.

$$t(\vec{x}) \equiv \frac{\partial u(\vec{x})}{\partial n(\vec{x})} = 0, \quad \vec{x} \in S \text{ and } B \quad (2)$$

where $\partial / \partial n$ denotes the normal derivative and the shear modulus $\mu$ is chosen as one for simplicity.

To construct the integral equation, we use the Green’s function, which satisfies the equation of motion and the boundary condition as follows:

$$\nabla^2 G(\vec{x} - \vec{y}) + k_T^2 G(\vec{x} - \vec{y}) = -\delta(\vec{x} - \vec{y}), \quad (3)$$

$$\partial G(\vec{x} - \vec{y}) / \partial n(\vec{x}) = 0, \quad \vec{x} \in B \quad (x_2 = \pm h). \quad (4)$$

The total wave field $u$ can be represented as a sum of the incident wave $u^{\text{in}}$ and the scattered wave $u^{\text{sc}}$. Applying the reciprocal theorem to the scattered wave $u^{\text{sc}}$ and the Green’s function $G$, we have the integral expressions for the displacement $u^{\text{sc}}$ and the traction $t^{\text{sc}}$ of the scattered wave.

$$u^{\text{sc}}(\vec{x}) = \int_{S} \frac{\partial G(\vec{x} - \vec{y})}{\partial n(\vec{y})} [u(\vec{y})] ds_y, \quad \vec{x} \in D \quad (5)$$

$$t^{\text{sc}}(\vec{x}) = \frac{\partial}{\partial n(\vec{x})} \int_{S} \frac{\partial G(\vec{x} - \vec{y})}{\partial n(\vec{y})} [u(\vec{y})] ds_y, \quad \vec{x} \in D \quad (6)$$

where $[u]$ is the crack opening displacement. It is noted that the above integral equations involve the integration over the crack face $S$ only, because both $t^{\text{sc}}$ and $G$ satisfy the traction free condition on $B$ as seen in eqs.(2) and (4). Taking the limit of $\vec{x} \in D \rightarrow \vec{x} \in S$, and substituting eq.(6) into the boundary condition (2), namely,

$t(\vec{x}) = t^{\text{in}}(\vec{x}) + t^{\text{sc}}(\vec{x}) = 0, \vec{x} \in S$, the boundary integral equation is obtained in the following form:

$$p.f. \frac{\partial}{\partial n(\vec{x})} \int_{S} \frac{\partial G(\vec{x} - \vec{y})}{\partial n(\vec{y})} [u(\vec{y})] ds_y = -t^{\text{in}}(\vec{x}), \quad \vec{x} \in S \quad (7)$$

where $p.f.$ indicates the finite part of the integral.