3. ADDITIONAL TOPICS IN PRODUCTION ECONOMICS

3.1 Introduction

The production economics concepts discussed in Chapter 2 should provide sufficient background for many of the basic efficiency and productivity measurement methods discussed in this book. Much of the Chapter 2 material is similar to what one would be likely to encounter in an undergraduate microeconomics course. The present chapter reviews some additional, more advanced, production economics material. Our primary focus in this chapter is on duality and multiple-input, multiple-output distance functions. An appreciation of the material in this chapter will ensure a deeper understanding of the basic productivity and efficiency measurement methods, and will also assist with the interpretation of more advanced methods, such as the measurement of allocative efficiency using stochastic frontier cost functions, which is discussed in Chapter 9.

This chapter is organised as follows. In Section 3.1 we discuss the duality between the production function and cost and profit functions and their properties. Following this we discuss econometric estimation of cost and profit functions in Section 3.2 and then introduce multi-output technologies and distance functions in Section 3.3. The chapter ends with some concluding remarks in Section 3.4.

3.2 Duality in Production

In this section we discuss the dual relationship between the production function representation of a production technology and the minimum cost or maximum profit
function representations of a production technology.\(^1\) It should be noted that the term *duality* is not used solely in production economics. It is also used in many other areas of study to describe two methods of approaching the same question. For example, it is used in consumer theory to describe the relationship between utility functions and expenditure functions.\(^2\) It is also used in linear programming to describe how any maximisation (minimisation) problem can be expressed as an equivalent minimisation (maximisation) problem.

The discussion of duality in production economics (hereafter termed simply “duality”) in this chapter is based primarily upon Chapter 6 in Beattie and Taylor (1985) and to a lesser extent upon Chambers (1988). Further relevant readings are listed in the final section of this chapter.

### 3.2.1 A Digression on Input Demand and Output Supply Functions

Two concepts that should be defined prior to our discussion of duality are those of input demand equations and output supply equations. We did not consider these concepts in Chapter 2 but came very close to doing so. We assumed that prices (both for inputs and output) were fixed at certain values.\(^3\) Hence, when we used both graphs and calculus to obtain the quantities of inputs and output that maximise profits, we had, in fact, obtained the quantities corresponding to one point on each of:

- the output supply curve;
- the labour input demand curve; and
- the capital input demand curve.\(^4\)

That is, we had identified those points corresponding to the price levels we had assumed.

We now describe how these profit-maximising output supply and input demand functions can be derived using either graphs or calculus. Following this we will also describe how conditional input demand curves can also be derived when cost minimisation is assumed.

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\(^1\) One may also represent a production technology in other ways, such as by a revenue function or by a cost indirect revenue function or a revenue indirect cost function, however, the cost and profit approaches have been the ones most often considered. For a comprehensive discussion of various dual production representations and their inter-relationships see Fare and Primont (1995).

\(^2\) Comes (1992) provides an excellent discussion of the many uses of duality in economics.

\(^3\) Recall our example involved two inputs (labour=\(x_1\), capital=\(x_2\)) and one output (\(y\)), with prices \(w_1\), \(w_2\) and \(p\), respectively. In this Chapter we will assume we are dealing with the long run and hence that all inputs are variable. Results involving one or more fixed factors can also be derived, but these would be more messy than the case where all inputs are variable. All assumptions made in Chapter 2 are also made in this chapter, unless otherwise stated.

\(^4\) Note that these curves represent the quantities supplied and demanded by a single firm not those of the entire industry.