5. ECONOMIC THEORY AND INDEX NUMBERS

5.1 Introduction

This chapter is primarily devoted to a detailed examination of the economic-theoretic foundations of the various index numbers discussed in Chapter 4. Given the importance attached to index numbers in TFP measurement, it is hardly surprising that economic theory is extremely relevant in understanding what these index numbers actually measure and in making a proper application of the formulae described. This chapter is also important in that it provides a base from which we integrate the three principal approaches, viz., the index number, DEA and stochastic frontier approaches, in the context of productivity and efficiency measurement.

The economic-theoretic approach to index numbers is also known as the functional approach to index numbers, since the approach postulates a functional relationship between observed prices and quantities for inputs as well as outputs. In the case of productivity measurement, the economic theory relevant to production (i.e., the microeconomic theory of the firm) is even more relevant. The functional approach contrasts with the simple mathematical approach, usually known as the test (or axiomatic) approach (considered in Chapter 4) and its reliance on a range of well-defined properties or tests or axioms.

In this chapter we make use of the production theory and duality discussed in Chapters 2 and 3. It is inevitable that the approach explored here appears more involved and difficult, but we suggest that the reader focuses on the final results, their interpretation and their implications for the choice of a formula. A major feature of this chapter is that, despite the complexities of the theory involved, most
discussions ultimately lead to or recommend the use of the Tornqvist or Fisher formulae for measuring price and quantity changes and in deriving a measure of total factor productivity. Thus, it is reassuring to note that these two formulae, which possess nice statistical properties, and which are computationally very simple, also possess some attractive economic-theoretic properties.

The approach we examine revolves around two basic indices proposed decades ago. All price index numbers, input as well as output price indices, are based on the famous Konus (1924) index and the approach discussed in Fisher and Shell (1972). The Konus index, in its original form, provided an analytical framework for measuring changes in consumer prices, i.e., the construction of cost of living index numbers. The input and output quantity index numbers, and productivity indices, are all based on the ideas of Malmquist and the distance function approach outlined in Malmquist (1953). No treatment of these aspects would be complete without reference to work by Dievert (1976, 1978, 1981), Caves, Christensen and Dievert (1982a and 1982b), Färe, Grosskopf and Lovell (1985, 1994), Färe and Primont (1995) and Balk (1997).

The plan for the chapter is as follows. Section 5.2 presents a simple case of one output and one input and uses it to examine the TFP index developed in Chapter 4 in more detail in conjunction with a production function. This section provides a simple decomposition of the TFP index from an economic-theoretic angle, and it helps in integrating the TFP index with some of the more recent work on efficiency and productivity measurement using Malmquist DEA and other techniques, when panel data are available. In Section 5.3 we examine the framework for defining input and output price index numbers, and see where the price index numbers defined in Chapter 4 fit into this approach. Section 5.4 focuses on input and output quantity index numbers, and Section 5.5 describes Malmquist productivity index numbers. The final section brings the various strands and approaches together, and provides a lead into the two principal frontier measurement techniques discussed in Chapters 6 to 9.

5.2 Decomposition of a Simple TFP Index

Let us consider the TFP index in the simplest case of a single input and a single output. Let \( y_t, y_s \) and \( x_t, x_s \) represent observed quantities of outputs and inputs produced by a firm in periods \( t \) and \( s \), respectively. Suppose the production technologies in these two periods are represented by functions, \( f_t(x) \) and \( f_s(x) \).

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1 The analysis here is based on Balk (1997), in which this simple case is used in making a case for direct and indirect quantity index numbers in productivity measurement.
2 A similar analysis holds when \( s \) and \( t \) represent two firms, \( s \) and \( t \), located in different geographic locations in a given time period.
3 We assume that these functions possess characteristics associated with production functions derived from a production technology satisfying standard axioms, discussed in Chapter 3.