10 A SURVEY OF INTERVAL CAPACITY CONSISTENCY TESTS FOR TIME- AND RESOURCE-CONSTRAINED SCHEDULING

Ulrich Dorndorf
Toàn Phan Huy
Erwin Pesch

University of Bonn, Faculty of Economics, BWL 3
Adenauerallee 24-42, D-53113 Bonn, Germany
udorndorf@acm.org

10.1 Overview

Interval capacity consistency tests consider the resource capacities available and required within certain time intervals. The goal of the tests is to draw conclusions that allow to rule out inadmissible activity start times or sequences. The tests can be effectively used to reduce the search space of difficult time- and resource-constrained scheduling problems. They have successfully been applied in algorithms for solving idealised problems such as the classical job shop scheduling problem (JSP) or the resource-constrained project scheduling problem (RCPSP) as well as for solving industrial scheduling problems. For instance, it seems fair to say that the advances in modern branch and bound algorithms for the JSP that have been made in the last decade can to a large extent be attributed to the effect of interval consistency tests, some of which are also known under the names of immediate selection, edge finding, and energetic reasoning. The tests can also serve to derive tight lower bounds for makespan minimisation problems.

This chapter presents interval consistency tests for disjunctive and general resource-constrained (cumulative) scheduling within a unified framework, using numerous examples for illustration. We review the state of the art and derive some new results for disjunctive scheduling.
10.2 Introduction

Time and resource constrained scheduling is concerned with the task of scheduling a number of activities subject to temporal constraints between activities, such as precedence or synchronisation, and constraints on the availability of several shared resources. An activity \( i \) is characterised by its processing time \( p_i \) and resource requirements \( r_{ik} \): It requires \( r_{ik} \) units of a renewable resource \( k \) for each of \( p_i \) time units, and it releases the resource units again upon completion. Once begun, an activity must be processed in an uninterrupted fashion. A resource \( k \) is available in constant amount \( R_k \).

By imposing an upper bound on the latest completion time and a lower bound on the earliest start of all activities, activity start time windows can be derived in a straightforward way. The start time window of activity \( i \) is the interval between the earliest start time \( est_i \) and the latest start time \( lst_i \) of \( i \). The domain \( \delta_i \) is the set of all possible start time assignments \( st_i \) of \( i \). It is bounded by the start time window; because some values in the start time window may be excluded, e.g. by temporal constraints of the type \( st_i \neq t_x \), for some time \( t_x \), we can state in general that \( \delta_i \subseteq [est_i, lst_i] \).

The purpose of this chapter is to present a class of logical tests called interval capacity consistency tests which are based on resource constraints. These tests allow to reduce activity domains by ruling out infeasible start time assignments. They can be applied in scheduling algorithms such as list scheduling or branch and bound procedures, or in constraint propagation based scheduling systems. The benefit of the tests is that they can reduce the search space and direct an algorithm towards good solutions. Here, we are only interested in the tests themselves and do not address scheduling algorithms in which they can be embedded. Since the tests only eliminate solutions incompatible with the capacity constraints, they are independent of the overall objective function to be optimised. The assumptions that we have made so far are rather general and cover \( \mathcal{NP} \)-hard models such as the classical job shop problem (JSP) (Błazewicz et al. 1996a,b) and the resource-constrained project scheduling problem with simple (RCPSP) or generalised precedence constraints (RCPSP/max) (Brucker et al. 1998, Domschke and Drexl 1991). In models with generalised precedence constraints time varying resource supply can easily be reflected by introducing dummy activities (Bartusch et al. 1988).

In the following sections we assume that all activity domains have been made consistent with the temporal constraints. Efficient algorithms for this are well known; a detailed description of this process in a constraint propagation based scheduling system is for instance given by Nuijten (1994). We are interested in further reducing the domains by applying interval consistency tests.

In the literature, activity domains are often approximated by start time windows, and this approximation is then referred to as activity release times and due dates, or heads and tails. The domain reduction process may then be called adjustment of heads and tails or time bound adjustment. Specific interval consistency tests have become known under the names immediate selection, edge finding, and energetic reasoning.

The remainder of this chapter is organised as follows. Section 10.3 introduces some additional formal notation and a graphical representation used for the examples