RESEARCH IN STATISTICAL TOLERANCING: 
EXAMPLES OF INTRINSIC NON-NORMALITIES, AND THEIR EFFECTS

by

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ABSTRACT: This paper summarizes the results of exploratory studies of the statistics of actual values (minimal enclosing zones) of essentially two-dimensional geometric position, circularity, and runout tolerances. Data for the studies were collected via CMM measurements of 100-sample lots of commercial washers and automotive valve seats. Phenomenological models were devised to predict the measured statistics, and agree reasonably well with the experimental data.

The metric used to assess the effects of non-normality is failure rate, i.e. the probability that a part selected randomly from a population will fail to meet a fixed tolerance specification. Significant differences are noted in the failure rate estimated for features exhibiting the modeled, non-normal statistics versus those having normal statistics. The paper shows that blind use of techniques that rely on normal statistics, such as those based on the Cp, Cpk process control indices, can be dangerous when the actual statistics are not normal.

KEYWORDS: dimensional tolerance, statistical tolerance, geometric tolerance, non-normal statistics

1 INTRODUCTION

Statistical tolerancing has evolved over forty years [Brooks 56] as a means for using less restrictive, and thus less costly, tolerances on parts used in many types of assemblies. As currently formulated, statistical tolerancing techniques apply mainly to parametric (dimensional limit, or 'plus/minus') tolerances, and rely heavily on dimensional variations being independent and normally distributed. A simple example: the position of the hole in Figure 1-1a is controlled by dimensions with worst-case limits. A statistical model can be superimposed on a worst-case design by assuming, as in Figure 1-1b, that the horizontal position of the center, xc, is a random variable whose probability density function (pdf) is N(μ, σ²), i.e. a normal (Gaussian) density with mean value μ and variance σ². A similar assumption is made for yc, the vertical position of the center, and - critically - xc and yc are presumed to be statistically independent.

The tails of the N(μ, σ²) densities that lie outside of the worst-case limits correspond to defective parts. However, when xc and yc are components of linear assembly dimension chains, statistical averaging comes into play and parts that are defective on a worst-case, part-level basis may be acceptable as components of statistically controlled assemblies.

Figure 1-1: Some parametric tolerances, and a statistical interpretation thereof.

1 The tails of the N(μ, σ²) densities that lie outside of the worst-case limits correspond to defective parts. However, when xc and yc are components of linear assembly dimension chains, statistical averaging comes into play and parts that are defective on a worst-case, part-level basis may be acceptable as components of statistically controlled assemblies.
Parametric tolerancing, as in Figure 1-1a, suffers from various well known ambiguities and has been gradually replaced in many mechanical applications by geometric tolerancing. However, little is known about the statistical properties of variations controlled by geometric tolerances. Figure 1-2a shows a geometrically tolerated version of Figure 1-1a. The meaning of the position tolerance (the $\Phi$ block in Figure 1-2a) is shown in Figure 1-2b. The axis of the hole must lie within a cylindrical zone of diameter 1.0, that is normal to datum A and centered on the 'True Position' specified by the Basic (boxed) dimensions. The point labeled 'Actual Position' denotes the (measured) axis of an actual hole that happens to be normal to the A-datum induced from the measured part. The actual value of the position deviation for the measured part is the diameter of the smallest zone that contains the measured axis. When the feature's actual value is less than or equal to the specified tolerance value, the feature conforms to the tolerance specification.

Now, what kinds of statistical statements - perhaps relating to conformance - can we make about the variates associated with geometric tolerances?

The following experiments were designed to test models describing the statistical properties of actual values for position, circularity, and runout tolerances. Data were obtained by measuring populations of mass-produced commercial parts, and experimental distributions computed; these are compared below to distribution models.

Our approach was (1) devise a mathematical model from phenomenological reasoning; then (2) compare predictions from the model to the measured data. An alternative would have been to measure first, then look for the family of distributions that best fit the data. We conclude that the 'natural' statistics for some geometrically tolerated attributes are not normal, and this has important implications for some established quality criteria, such as the process capability indices $C_p$ and $C_{pk}$.

2 EXPERIMENTAL DESIGN

The goal of our experimental probe was to obtain statistical data for geometric tolerances. The first step was to select a population of parts whose dimensions are controlled with geometric tolerances. After obtaining parts, a measurement plan was developed to assess actual values for a subset of selected geometric tolerances. This plan encompassed both the collection of data and the selection of algorithms for data reduction.

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2 The most recent release of the ANSI dimensioning and tolerancing standard [Y14.5M 1994] contains the symbol $\text{ST}$ to identify statistical tolerances. The standard does not define what the symbol means (does not define 'statistical tolerance'), but it requires that statistically tolerated features be produced using statistical process controls. This requirement verges on inconsistency with Sect. 1.4e of Y14.5M (process independence).

3 'Actual value' is a relatively new concept in geometric tolerancing first formalized in the 'mathematical definitions' standard [Y14.5.1M 1994], which is a companion to the main Y14.5 standard.