Example: Alloy solidification

18.1 THE SCOPE OF THIS CHAPTER

The fabrication of shaped articles of metals or metal alloys often involves a solidification process, in which a molten metal or metal alloy is charged to a cold mold and then frozen. A significant difference between the solidification behavior of metals and that of metal alloys is well known, however, in that the solid-liquid boundary in the former is often regular in shape and occupies a relatively thin region at a given instant, while that of the latter may be highly irregular and may occupy a relatively thick region within the mold cavity. In fact, in the solidification of molten alloys, three regions typically may co-exist during the intermediate stages of the process: a (single-phase) solid region adjacent to the wall of the mold, a (single-phase) liquid region in the central portion of the mold, and a (two-phase) solid-liquid transition region between the two. In this case, the solidification takes place in this two-phase (or so-called "mushy") region at the interface between solid particles and the liquid that envelopes them, and may also take place at the boundary that the single-phase solid region shares with the "mushy" region. Moreover, since these two kinds of solidification occur in different regions within the mold cavity, a spatially non-uniform microstructure may be formed as a result (which in turn is likely to give rise to markedly non-uniform material properties in the final article).

A large body of empirical information exists, of course, concerning the effect of solidification conditions on the microstructure of alloys cast in this manner [1-3]; on the other hand, a more detailed and quantitative understanding of the process can of course be developed using, for example, the appropriate multiphase balance equations of Chapter 4 and the pertinent constitutive relationships of Chapters 8-10.

The microstructural variations that arise are due to the presence of time-varying solidification conditions. That is, initially and near the mold's wall, the cooling rate in that region is relatively high and a layer of single-phase alloy of relatively uniform thickness is formed. Thereafter, however, the cooling rate becomes somewhat lower, giving rise to a region of finite thickness, immediately adjacent to the just-solidified wall layer, in which a two-phase liquid-solid dispersion is the thermodynamically stable state (at least instantaneously and locally). Furthermore, when this "mushy" region is formed, a relatively large increase in the interphase boundary area may be realized, so that the interphase transfer of the mass of liquid-phase components may proceed at a more rapid rate. In fact, this interphase transfer of mass in the "mushy" zone may occur so rapidly that the region near a given interphase boundary surface may become relatively depleted with respect to one of the diffusing liquid-phase components: the rate of interphase transfer then becomes a diffusion-limited one. In addition, gradients in both the temperature and the mass density of a liquid-phase component may give rise to buoyancy-driven flow of liquid in the single-phase liquid region (and...
possibly through the two-phase region as well). It may thus be seen that each of these factors may give rise to spatially non-uniform cooling and solidification conditions, which in turn can give rise to (sometimes significant) microstructural variations within the final article.

In this chapter, the development of a theoretical model of the solidification of a two-component molten alloy will be undertaken. In doing so, the energy and mass transfer, single- and two-phase fluid flow, and interphase mass and momentum exchange processes—that is, all of the relevant physical and chemical phenomena—will be incorporated. In addition, the necessary balance equations, constitutive relationships, appropriate simplifications, initial and boundary conditions, material property and process parameter values, a numerical solution technique, and typical output quantities will be described.

18.2 BALANCE EQUATIONS REQUIRED

In this solidification process, consider a eutectic-forming liquid mixture that consists of two-components ("A" and "B") and that resides within a mold which is rectangular in cross-section and relatively much longer in the third direction. Three of the walls of the mold are maintained at a temperature below the eutectic temperature of the mixture, while the fourth wall is thermally insulated. See also Fig. 18-1. Solidification begins locally when the temperature of the mixture falls below that of the solidus line for that composition, and may involve the addition of liquid-phase components to an existing planar solidification front, or may involve the nucleation and growth of the solid-phase particles that are dispersed in the liquid phase. Flow of the liquid phase may occur as a result of pressure gradients or of buoyancy forces, and may occur in the single-phase liquid region, as well as the "mushy" region (that is, when the liquid-phase volume fraction is not too low). Motion of the solid phase in the "mushy" zone may also occur—that is, when the volume fraction of solid phase is low and its particles are discrete—as its particles are swept along by the flowing liquid phase; when the volume fraction of the solid phase in the "mushy" zone becomes sufficiently high, however, the solid phase becomes a rigid porous solid and its velocity falls to zero. The only body force that acts on the solidifying material in the mold cavity is of gravitational origin, while neither surface stresses nor couple-stresses (of either mechanical or electromagnetic origin) act on the system. As a result, it may be seen that this is a two-phase, two-component system, from which thermal energy is being withdrawn, and in which momentum, mass and energy is being redistributed (both within and between phases).

Several balance equations are required for this theoretical model of the alloy solidification process. First, the balance equation for the total mass density of the liquid phase can be taken directly from Eqn. (12.2.2), with Table 12-1, as

\[ \frac{\partial \langle \rho^L \rangle}{\partial t} = -\nabla \cdot \langle \rho^L \rangle \mathbf{v}_L - \langle I_{p(\text{LS})} \rangle^L \]  

(18.2.1)

in which \( \langle \rho^L \rangle \) denotes the intrinsic volume average of the total mass density of the liquid phase in the (multiphase) material, \( \mathbf{v}_L \) is the velocity of the center-of-mass of the liquid phase, and \( \langle I_{p(\text{LS})} \rangle^L \) is the rate, per unit volume of the (multiphase) material, that material leaves the liquid phase ("L") and enters the solid phase ("S")