Chapter 3

A Note on Merton’s “Optimum Consumption and Portfolio Rules in a Continuous-Time Model”

(with M. I. Taksar)


S. P. Sethi, Optimal Consumption and Investment with Bankruptcy
1. Introduction

Notes, Comments, and Letters to the Editor

A Note on Merton's "Optimum Consumption and Portfolio Rules in a Continuous-Time Model"*

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In the paper "Optimum Consumption and Portfolio Rules in a continuous-Time Model," by R. C. Merton (J. Econ. Theory 3 (1971), 373–413), solutions obtained in cases when marginal utility at zero consumption is finite are not feasible. While they do satisfy the Hamilton–Jacobi Bellman equations, they do not represent appropriate value functions because the boundary behavior near zero wealth is not satisfactorily dealt with. In this note, we specify the boundary behavior and characterize optimal solutions. Journal of Economic Literature Classification Numbers: 022, 213. © 1988 Academic Press, Inc.

1. INTRODUCTION

In the area of consumption and portfolio problem in continuous time, Merton [2] is the most widely cited paper. It is an important paper because of its many significant contributions. Among these was the provision of explicit solutions for utility functions in the HARA family specified in Eq. (43) of Merton [2]. These solutions in the form of lengthy formulas were simply stated without any derivation. These contain errors that are perhaps difficult to detect because of the lack of derivation. While some minor errors were corrected in Merton [3], some significant errors, of which many investigators citing Merton [3] seem to be unaware,

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