The problem of localization of light in a random atmosphere requires, for comparison, a knowledge of the classical transport of light by an isotropically scattering medium in a bounded medium such as a slab. [See V. Sobolev, *Light Scattering in Planetary Atmospheres*, Pergamon (1975)] We consider the time-dependent Milne problem generalized to a slab with absorbing boundary conditions. We can adapt results obtained for phonon transport in GaAs at low temperatures to the corresponding optical problem. Transport of a pulse across the slab is evaluated by Monte Carlo methods, as well as by analytic approximations. Although the behavior is diffusive, when the detector is moved transversely, the mean time of arrival is found to be linear in the distance from point source to point detector!

1. INTRODUCTION

Recent efforts on electron localization in random systems have stimulated proposals that photon localization can also be achieved. Preliminary measurements exhibiting weak localization have been made in essentially classical systems. In order to be sure one has observed localization, it is necessary to know what photon transport is to be expected when localization is absent.

Because almost no analytic solutions exist to most transport problems, one often resorts to Monte Carlo calculations to obtain insight. We have recently performed a series of Monte Carlo calculations on transport in a more complicated system—phonons at low temperatures in GaAs. These calculations permitted the possibility of down-conversion i.e. break-up of a phonon into two lower frequency phonons. In order to detect, and remove, possible errors in the Monte Carlo code we needed a special case to check, in which theoretical estimates were available.

The check calculations performed were on a system in which down-conversion was prevented and isotropic, elastic scattering is all that is permitted. To our chagrin, even for this simple, special case no analytic solution was available. However, by extending the concept of *extrapolation length* that arose out of an analysis of the Milne problem in astrophysics (light scattering in a semi-infinite atmosphere) and was heavily exploited in neutron transport theory we found an approximate solution to the transport of particles in a slab with absorbing boundaries.
Some rather startling Monte Carlo results were found, and confirmed by our semi-analytic procedures. We shall report these results here because they are equally applicable to photon transport.

2. DESCRIPTION OF THE PROBLEM

We consider a slab whose walls are perfectly absorbing to photons hitting them from inside. Photons hitting the left wall are absorbed. Those hitting the right wall are "detected" and absorbed. In between the walls, isotropic scattering takes place. A photon pulse (a delta function in time) enters the slab at a point from the left. We would like to obtain the shape and normalization of the emerging pulse at any point on the right hand surface. The shape of this pulse is a solution of the notoriously difficult first passage time problem!11 The time integral over the pulse and over some area of the right hand surface indicates the total number of photons that would be counted by a detector covering that area. If the area covers the entire right hand surface, the integral determines the total number of photons that survive the passage across the slab. If the results are normalized to one input photon traveling toward the right (at a random angle) we obtain the fraction, \( F \), of photons that survive the passage across the slab.

A second objective is the dependence of the flux on the transverse radial distance \( R \) or the actual distance \( r = \sqrt{R^2 + w^2} \) from the point of origin, where \( w \) is the slab width.

A third objective is the dependence of the peak or the mean time of arrival, \( t \), on the radial distance \( r \) from source to detector. The time \( t \), in random process literature, is known as the mean first passage time.

3. SURPRISING RESULTS

A. The Fraction of Surviving Photons

The fraction of photons that survive a passage across the slab is clearly a function, \( F(w/\lambda) \), of the ratio of the slab thickness \( w \) to the mean free path \( \lambda \). Moreover, \( F(0) = 1 \) and \( F(\infty) = 0 \). But how does the decrease take place? The most natural choice obtained by analogy from other areas of physics would be \( F(w/\lambda) = \exp(-w/\lambda) \). For \( w/\lambda \geq 1 \), our modified diffusion approximation yields:

\[
F(w/\lambda) = \frac{.8126}{1 + .7034(w/\lambda)}
\]  

(3.1)

See Table I for a comparison with Monte Carlo results.

<table>
<thead>
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<th>( w )</th>
<th>( \sigma )</th>
<th>( \lambda )</th>
<th>( w/\lambda )</th>
<th>( w/\lambda, F )</th>
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<th>( w/\lambda )</th>
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<td>( 6.182 \times 10^{-4} )</td>
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<td>.708</td>
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<td>.0945</td>
<td>.704</td>
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<td>.249</td>
<td>.699</td>
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<td>( 3.833 \times 10^{-2} )</td>
<td>.2</td>
<td>5.218</td>
<td>.1747</td>
<td>.701</td>
</tr>
</tbody>
</table>

\( w \) and \( \sigma \) refer to phonon calculations, and are used here only to correlate information in Table I with that in Table II.