1. Summary and introduction

Let \((T, \mathcal{A}, P)\) be a \(P\)-complete probability space, \(\mathcal{B}\) a \(P\)-complete \(\mathcal{A}\)-subfield of \(\mathcal{A}\) and \(X\) a separable locally convex Fréchet space. In the paper we prove that if a strongly \(\mathcal{A}\)-measurable function \(x(\cdot)\) with values in \(X\) is a selector of a random closed convex set \(C(t)\) which varies in a \(\mathcal{B}\)-measurable manner, then the same holds true for any version of its conditional expectation \(E^{\mathcal{B}}x(\cdot)\) (Theorem 1(i), Section 3). A version of conditional expectation can run, on some \(\mathcal{B}\)-measurable subset of \(T\), along extremal points of \(C(t)\) only if it coincides on this subset with the original function (Theorem 1 (ii), Section 3).

These results are used to obtain an extended version of Jensen's inequality for a random convex function \(f(t, \cdot)\) defined on a random convex subset \(S(t)\) of a separable locally convex Fréchet space \(V\), where the convexity of \(f\) is with respect to a random convex cone \(K(t)\) in a separable locally convex Fréchet space \(U\). The only essential assumptions needed here are \(\mathcal{B}\otimes\mathcal{B}(V \times U)\)-measurability of the epigraph of \(f\) and the closedness of its \(t\)-sections. If \(f(t, \cdot)\) is
strictly convex, then the obtained Jensen's inequality is also strict (Theorem 2, Section 4).

If the original function \( x(\cdot) \) is a selector of the random convex set on a subset \( A \) of \( T \), where \( A \in \mathcal{A} \), then any version of \( E^{B_A(\cdot)X(\cdot)}/E^{B_A} \) is a selector of \( C(\cdot) \) on the smallest \( \mathcal{B} \)-measurable set containing \( A \). This function can run, on an \( \mathcal{B} \)-measurable subset of \( A \), along extremal points of \( C(t) \) only if it coincides on this subset of \( A \) with the original function \( x(\cdot) \) (Theorem 1', Section 3).

The obtained results described above are analogous to those obtained by Pfanzagl [8] and Daures [2]. Pfanzagl considered the particular case when \( X = \mathbb{R}^n \) and \( C(t) = C \) is a constant Borel subset of \( \mathbb{R}^n \). Daures proved the corresponding results assuming that \( C(\cdot) \) is a \( \mathcal{B} \)-measurable multifunction such that \( C(t) \) does not contain lines and is a weakly locally compact and convex subset of a locally convex separable Fréchet space \( X \). In the case when \( X \) is a Banach space this result can also be derived from a result of Hiai and Umegaki [5; Theorem 5.3.10] provided \( C(t) \) is contained in a ball with a varying and integrable radius, or, implicitly, if \( C(\cdot) \) contains \( \mathcal{B} \)-measurable Bochner integrable selectors.

We are interested here in the case when \( X \) is an infinite dimensional separable locally convex Fréchet space and \( C(\cdot) \) is a convex closed \( \mathcal{B} \)-measurable multifunction. The aim of this paper is to prove that the Daures's theorems remain valid without the restrictive and inconvenient assumption that \( C(t) \) is weakly locally compact and does not contain lines. Our proof is, however, completely different from that of Daures since the method he used is based on a Klee-Olech characterization of convex sets which is useless in the present framework.