A Conservation Law Model for Ion Etching for Semiconductor Fabrication

In sections 12.1, 12.2 we shall describe a 2-dimensional mathematical model for ion etching which was presented on February 12, 1988 by David S. Ross from Eastman Kodak Company, and is based on his papers [1] [2]. In Section 12.3 several mathematical problems will be posed.

12.1 Etching of a Material Surface

A material with constant molecular density $N$ occupies space $\{0 < y < y(x, t)\}$. It is etched at its surface $y = y(x, t)$ by bombardment with beam of ions with constant flux density $\phi$ (see Figure (12.1)). The surface evolves according to [3]

\[
y = -\frac{\phi}{N} S(\theta)
\]

FIGURE 12.1.
where \( \theta = \arctan y_x(x,t) \) and \( S(\theta) \) is a function which depends on the material. The function \( f(p) = \frac{\delta}{F} S(p) \) is called the sputtering function; it is positive and uniformly bounded for \(-\infty < p < \infty\); it has several inflection points (see Figure 12.2).

![Figure 12.2](image)

The problem is to find the evolution of the etched surface, i.e., to solve the equation

\[
y_t + f(y_x) = 0
\]

subject to

\[
y(x,0) = y_0(x)
\]

If we differentiate (12.1) in \( x \) and set \( y_x = p \), then the problem reduces to solving the conservation law

\[
p_t + (f(p))_x = 0
\]

subject to

\[
p(x,0) = p_0(x),
\]

where \( p_0(x) = y_0'(x) \).

Introducing the characteristic curves, i.e., the solutions of

\[
\frac{dx}{dt} = f'(p(x,t))
\]

(assuming \( p \) is known) we compute, formally,

\[
\frac{dp(x(t),t)}{dt} = p_t + p_x \frac{dx}{dt} = 0,
\]