A LINEAR FILTER FOR DISCRETE SYSTEMS WITH CORRELATED MEASUREMENT NOISE

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ABSTRACT

This paper introduces an optimal linear filter for discrete systems with correlated measurement noise by generalized least square method which is novel in its structure, its derivation and its simplicity. The equations reduce to the standard Kalman filter equations when the measurement noise is independent. The new filter avoids the increased order and other complexities of previously proposed methods particularly those based on augmented state and differencing approaches.

I. INTRODUCTION

The mathematical solution to the problem of linear autonomous discrete time filters for correlated noise has been known since 1960, see Kalman [1]. His method, known as the state augmentation method, involves the construction of a shaping filter with independent noise input. The dynamics of the shaping filter are included as a part of the system dynamics, and its states are estimated.

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along with the system states. In practice, state augmentation requires an increase in the state vector dimensions, such dimensional increase may be prohibitive because the computer storage space is limited. Also it has long been recognized that the numerical solutions of the Riccati equations of the augmented states filter are difficult to obtain because of the fact that this Riccati equation for the correlated noise case is not well conditioned and the solution can be a singular matrix.

Much effort has been expanded by many investigators to alleviate the ill-conditioned computations of the augmented state approach by reducing the number of state variables estimated and reduce the order of Riccati equation [2-6]. As pointed out by Bucy et al. [8], these papers have all had one idea in common, using differences (time derivatives in continuous time) of consecutive measurements. This results in a modified "measurement" equation which contains independent noise and hence allows the use of Kalman filtering but there remains a complex supplemental estimation problem. Bucy [11, Chapter 9] studied this problem; his approach is essentially based on Kalman's [1] orthogonal projections. In his derivation it is required to estimate both the states and the correlated measurement noises, then eliminating the estimate of the noises from the measurement equation. More recently, Johnson [7] obtained a filter for correlated measurement noise by solving the matrix Wiener-Hopf equations, which is not strictly optimal, yet the resulting equations are very complicated. Bucy, Rappaport and Silverman [8,9] analyzed the correlated noise filtering problem for time invariant systems by a mathematical analysis of invariant directions of the Riccati equation of the augmented states.

Here in this paper a filter which accounts for correlated measurement noise is derived in a way that requires neither the augmentation by a shaping filter of the filter dynamic model nor the concepts of differencing of successive measurements. The filter equations are obtained by generalized least square method to minimize the quadratic form of the measurement errors weighted by the covariance matrix of the measurement noises.* This is an extension of the weighted regression analysis used by the authors in [10]. The derivation is straightforward and simple. It is not an approximation. The filter equations degenerate to the basic Kalman filter equations when it is assumed that the measurement noise is independent. Judging from the wide applications of the Kalman filter, the filter equations derived here should be very useful.

* At least one unsuccessful effort to solve the problem by this type of approach has been reported in the literature [11, Appendix A].