RELATION OF WATER TRANSPORT TO WATER CONTENT IN SWELLING

BIOLOGICAL MEMBRANES

Joel L. Bert and Irving Fatt

Department of Mechanical Engineering, University of California, Berkeley, California 94720

ABSTRACT

In biological and some synthetic polymer materials, which swell on imbibing water (corneal stroma, costal and articular cartilage, skin, cellulose acetate, and polymethyl methacrylate), the phenomenological parameter, \( k/\eta \), relating rate of water flow to pressure gradient in Darcy's law \( q = -(k/\eta)(dP/dx) \) has been found to be a function of water content. \( \log k/\eta \) appears to be linearly related to \( \log H \), where \( H \) is grams of water/grams of dry swelling material. A pore model predicts an increase in "pore" radius with increasing hydration. The "pore" radius of the corneal stroma predicted by the model is in agreement with the radius calculated from light scattering and diffusion data.

INTRODUCTION

Most biological materials have a fixed water content in a living system. If these materials are dehydrated by artificial means they will re imbibe water to restore the normal water content. Some biological materials have a smaller water content \textit{in situ} than they can hold if simply immersed in water. The corneal stroma is a well-studied example of such a system. An "active" process is supposedly operating in the living system to keep the water content of the corneal stroma to its normal level.\(^1\)

Fatt and Goldstick\(^2\) have shown that the transient water imbibition process in swelling membranes can be described by an equation analogous to the transient diffusion equation. The Fatt
and Goldstick equation is
\[ \frac{\partial H}{\partial t} = D(H) \frac{\partial^2 H}{\partial \psi^2} \]  
where the terms of this and all subsequent equations are defined in the nomenclature section. Fatt\(^3\) has shown that the water transport coefficient \(D(H)\), in corneal stroma, is given by
\[ D(H) = \frac{k}{\eta} \frac{\varepsilon^2}{(\varepsilon + H)} \frac{dP}{dH} \]  
The flow conductivity term, \(k/\eta\), in equation (2) is the same term that appears in Darcy's law,
\[ q = -\frac{k}{\eta} \frac{dP}{dx} \]  
where equation (3) is for steady state flow of a liquid of viscosity \(\eta\) in a one-dimensional system.

Both transient and steady state studies of water flow in swelling materials (Mishima and Hedbys,\(^4\) Fatt,\(^3\) Bert and Fatt\(^5\)) show that \(k/\eta\) is a function of hydration of the material.

If the swelling material is assumed to consist of a matrix of solid material through which the water flows, then a pore model can be hypothesized and tested. In such a model, the pore radius will be a function of hydration.

**PORE MODEL**

If in an area \(A_T\) of a membrane there are \(n\) pores, all of radius \(r\) and length \(\ell\), then Poiseuille's law gives the flow rate as,
\[ Q = n \pi r^4 \frac{\Delta P}{8 \eta \ell} \]  
For the same membrane Darcy's law gives
\[ Q = k A_T \frac{\Delta P}{\eta L} \]  
Equating (4) and (5) and letting the tortuosity term:be \(\tau\), where \(\tau = \ell/L\), we obtain
\[ k A_T = n \pi r^4 / 8 \tau \]  
The volume of water in the membrane is