4 GEOMETRY AND LINEAR ALGEBRA


No beginner's course in mathematics can do without linear algebra. According to current international standards it is presented axiomatically. It is a second generation mathematical model with its roots in Euclidean geometry, analytical geometry, and the theory of systems of linear equations. This brings pedagogical difficulties. Beginners with a shaky background in geometry and algebraic computation who also have difficulties with abstractions are really not ripe for the study of linear algebra. On the other hand, there is no need to exaggerate the difficulties. The theory is very simple, has few theorems and is free from complicated proofs. It is also a must. Not being familiar with the concepts of linear algebra such as linearity, vector, linear space, matrix, etc., nowadays amounts almost to being illiterate in the natural sciences and perhaps in the social sciences as well.

We shall devote the first part of this chapter to the three sources of linear algebra: Euclidean geometry, analytical geometry, and systems of linear equations. After a section on matrices we then pass to linear algebra proper and its objects, linear spaces and linear maps between them.
The rest of the chapter deals with linear analysis, an extremely useful hybrid of algebra and analysis obtained by introducing the notion of length into the algebraic machinery. Linear analysis in linear spaces of infinite dimension, usually called functional analysis, is a successful twentieth century invention. We shall review some of its basic concepts and results including the spectral theorem for self-adjoint linear operators. It will be applied to the analysis of small vibrations of mechanical systems.

The reader should realize that this chapter covers a lot of ground and that it is not meant for quick consumption. In many places some previous experience of the subject is required.

4.1 Euclidean geometry

*History*

Euclid's *Elements* was written in Alexandria around 300 B.C. It was preserved in handwritten copies until printing started, around 1500. The oldest copies now in existence are from about 1000 A.D. (T. L. Heath's English translation from 1908, with Commentary, is available as a Dover publication.) Until the beginning of this century, the *Elements* was the textbook of mathematics in secondary school. The strength of this position and also Heath's conservative leanings are abundantly clear from his preface: "It is of course not surprising that, in these days of short cuts, there should have arisen a movement to get rid of Euclid . . . a rush of competitors anxious to be first in the field with a new text book on the more 'practical' lines which now find so much favour." The Swedish poet C. M. Bellman wrote

When I think of Euclid, even now
I have to wipe my sweaty brow,

an echo of the despair of many generations of schoolchildren.

Thirteen parts of the *Elements* have been preserved. The first four parts deal with triangles, parallelograms, and circles. They explain and prove well-known geometric theorems, for instance that the sum of the three angles of a triangle is two right angles, the theorem of Pythagoras, the theorem that a circular arc is seen under a constant angle from the remaining part of the circle. The proofs rely on propositions that are not proved—in our terminology, axioms or postulates. They are a mixture of general logical principles like "those which are equal to the same are mutually equal," and geometric propositions such as the famous axiom of the parallels. The latter can be formulated as follows. Through a point outside a straight line there passes precisely one straight line parallel to the first one, i.e., the two never meet however far out they are prolonged.