CHAPTER VI

Techniques of Differentiation and Integration

VI.1 A Look Back

In the last three chapters we have accumulated a rather large number of formulas for both differentiation and integration. It is the purpose of this chapter to bring these results together, and to present ways by which these results can be extended. It would therefore be helpful to place the results obtained so far in tabular form for easy reference. We list here the results for both derivatives and integrals, pairing them off where this is appropriate. The differential can be easily obtained from the derivative, and for that reason will not be listed in this table.

(1a) $D_x[cf(x)] = cf'(x)$

\[ \int cf(x)dg(x) = c \int f(x)dg(x) \]

(1b) $f(x)d[cg(x)] = c \int f(x)dg(x)$

(2a) $D_x[f(x) \pm g(x)] = f'(x) \pm g'(x)$

\[ \int [f_1(x) \pm f_2(x)]dg(x) = \int f_1(x)dg(x) \pm \int f_2(x)dg(x) \]

(2b) $f(x)d[g_1(x) \pm g_2(x)] = \int f(x)dg_1(x) \pm \int f(x)dg_2(x)$

(3) $D_x[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

(4) $D_x\left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

(5a) $D_x[f(x)] = f'(x)$

\[ \int f(x)dg(x) = \int f(x)g'(x)dx \]

(5b) $\int_a^b f(h(x))dg(h(x)) = \int_{h(a)}^{h(b)} f(y)dg(y)$

(6a) $D_x(c) = 0$

\[ \int 0dg(x) = C \]

(6b) $D_x(x) = 1$

\[ \int dg(x) = g(x) + C \]

(6c) $D_x(x^n) = nx^{n-1}$

\[ \int x^n dx = x^{n+1}/(n + 1) + C, n \neq -1 \]

330

R. L. Wilson, Much Ado About Calculus
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As you will agree, this is a rather impressive list. You should keep this list at hand, or better yet be able to derive each result. However, this list is deficient in some rather obvious places as well as in some places that are not so obvious. We do not have an integral of the tangent, cotangent, secant, or cosecant, for instance, and there are many other functions that you might think of for which we do not have integrals, such as \((1 - x^2)^{1/2}\). In the next section we will introduce the use of logarithms for the purpose of simplifying some differentiations and making possible some others. In the remaining sections of this chapter we will provide methods for filling in some of the gaps in integration, such as those referred to above. The fact that some of the results may be a bit less than obvious should not surprise you, however, for there are many integrals for which no one can find a formula. (There