3.1 Introduction

In this chapter we take up the study of the dynamics of a shallow, rotating layer of homogeneous incompressible and inviscid fluid. There are two purposes to our consideration of this physical system. It is first of all simple enough so that the issues raised by the problem of geostrophic degeneracy can be dealt with directly without the need to simultaneously treat the complexities of the thermodynamics of a density-stratified fluid. The first goal of the present chapter is to illustrate how the geostrophic approximation can be systematically exploited to produce a deterministic dynamical framework adequate for the calculation of motions of large time and space scales. Furthermore, the method of analysis to be presented also can be generalized to the study of thermodynamically active fluids. The key technique of the analysis is the formulation of a systematic approximation scheme in which the geostrophic approximation is merely the first step.

The second purpose of this chapter is to then use the model to study motions of atmospheric and oceanic relevance. Of course it is not clear ahead of time that a model which completely ignores the presence of stratification will be at all useful for this purpose. At this point, such a suggestion simply presents for consideration a model of the real atmosphere or ocean, whose validity must be examined after the fact. This step of modeling is always, to some degree or other, a leap of faith. Whether the step is a useful one depends on the intuitive skill of the modeler in anticipating the way the physical model shares the essential dynamical character of the
geophysical system. The step of choosing the model is informed by experience, personal intuition, and a degree of explicit dynamical reasoning. It is important to emphasize that once the model has been chosen the proper test of its usefulness can only come from the systematic, logical, and precise working out of the dynamical predictions of the laws of motion as they apply to the model.

Experience has shown that the shallow-water model is capable of describing important aspects of atmospheric and oceanic motions so that we can now directly take to our advantage the profound intuitive insights of earlier workers* in the field. Necessarily, the motions described by such a model can only be expected to apply to phenomena which do not depend in a crucial way on stratification.

### 3.2 The Shallow-Water Model

We start by considering a sheet of fluid with constant and uniform density as shown in Figure 3.2.1. The height of the surface of the fluid above the reference level \( z = 0 \) is \( h(x, y, t) \). With application to the earth’s atmosphere

![Figure 3.2.1 The shallow-water model.](image)

or ocean in mind, we model the body force arising from the potential \( \Phi \) as a vector, \( \mathbf{g} \), directed perpendicular to the \( z = 0 \) surface, i.e., antiparallel to the vertical axis. The rotation axis of the fluid coincides with the \( z \)-axis in the model, i.e., \( \Omega = k \Omega \), so that in this case the Coriolis parameter \( f \) is simply \( 2\Omega \). The rigid bottom is defined by the surface \( z = h_B(x, y) \). The velocity has components \( u, v, \) and \( w \) parallel to the \( x-, y-, \) and \( z \)-axes respectively. The pressure of the fluid surface can be arbitrarily imposed, but for our purposes we may take it to be constant. Finally, the fluid is assumed inviscid, that is,

* This intuitive leap is astoundingly clear in the work of Rossby (1939) and Stommel (1948).