COUPLED TRANSPORT OF O₂ AND CO₂ WITHIN THE UPPER SKIN SIMULATED BY THE CAPILLARY LOOP MODEL

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INTRODUCTION

The transport of respiratory gases as oxygen and carbon dioxide through tissues and blood and their functional relations has been investigated in the past by numerous authors (for example: Roughton, 1964; Severinghaus, 1966; Thews, 1968; Lübbers, 1979). Due to the introduction of transcutaneous Po₂ and PcO₂ measuring techniques in clinical medicine (Huch et al., 1981) the transport of oxygen and carbon dioxide through the skin has become of considerable interest. To study those processes in detail we developed a so-called capillary loop model, which simulates the gas and heat transport through the skin including the loop-shaped capillary structure of the upper skin and the nonlinear effects due to the binding of O₂ by hemoglobin and temperature shifts (Großmann et al., 1980; Großmann, 1982).

The purpose of this paper is to study the simultaneous transport of oxygen and carbon dioxide through the upper skin under various conditions using the capillary loop model.

The Model

That part of the skin we are interested in is shown on the schematic drawing of the left side of Fig. 1. It consists of the epidermis (E) and the neighbouring stratum papillare containing the characteristic capillary loops.

To analyze the combined transport of oxygen and carbon dioxide through this part of the skin we used the microcirculatory unit shown on the right side of Fig. 1 as a model. Its position is marked...
in the schematical drawing on the left side by μ. The unit consists of three spatial compartments, the dead part of the epidermis (ed), the viable part of the epidermis (ev), and the stratum papillare (sp) containing a single capillary loop. (ar) indicates the arterial inflow, (ve) the venous outflow of the capillary, and (cd) the capillary dome.

Assuming that a defined region of the upper skin is built symmetrically by several, equal microcirculatory units and assuming that diffusion conditions, oxygen consumption, carbon dioxide production, blood flow etc. are homogeneous, the lateral boundaries of each microcirculatory unit are surfaces of symmetry for $P_{O_2}$ and $P_{CO_2}$. Thus, the concentration gradients perpendicular to the lateral boundaries vanish. With Fick's law of diffusion (Crank, 1955) it follows that no diffusion flux occurs across these boundaries. This means that each μ is isolated from its neighbours and can be analyzed as a representative unit of this defined skin region.

In the tissue compartments, $O_2$ and $CO_2$ are transported by diffusion. The relationship of $O_2$ consumption and $CO_2$ production is given by the respiration quotient. In the capillary, $O_2$ and $CO_2$ are transported by diffusion and convection. "Blood" is assumed to be a homogeneous plasma-hemoglobin solution. Oxygen is physically dissolved in the plasma and chemically reversibly bound by hemoglobin. $CO_2$ is physically dissolved in the plasma, stored as bicarbonate and reversibly bound by hemoglobin carbamino compounds. Equilibrium is assumed for all reactions of $O_2$ and $CO_2$.

Transforming these assumptions into mathematical language yields the following steady-state equations for $P_{O_2}$ and $P_{CO_2}$:

$$0 = \nabla \left( \alpha_{O_2} \cdot D_{O_2} \nabla P_{O_2} - \nabla \cdot C_{O_2} \ (P_{O_2}, P_{CO_2}) \right)$$  \hspace{1cm} (1)

$$0 = \nabla \left( \alpha_{CO_2} \cdot D_{O_2} \nabla P_{CO_2} - \nabla \cdot C_{CO_2} \ (P_{O_2}, P_{CO_2}) \right)$$  \hspace{1cm} (2)

for the capillary compartment, and