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Measures of Possibility and Fuzzy Sets

The material in this book is based on a nontraditional approach to the imprecise and the uncertain. The basic concept is the measure of possibility. The object of this introduction is to provide motivation and context, to define measures of possibility, and to present basic notions necessary for understanding the later chapters. It appeals considerably to results contained in the authors' theses [3, 24], among other references.

1.1. Imprecision and Uncertainty

Imprecision and uncertainty can be considered as two complementary aspects of a single reality, that of imperfect information. We shall assume here that an item of information can be expressed as a logical proposition with predicates, and possibly quantifiers. A corpus of knowledge will be a collection of items of information possessed by an individual (or a computer system, or a group of individuals), relating to a single problem. The predicates that occur in the expressions of this information can then be interpreted as subsets of a single domain of reference. A proposition can also be considered as an affirmation concerning the occurrence of an event. Such events can themselves be represented as subsets of this reference domain, which can therefore be called “the sure event.” Thus we have three equivalent ways to envisage a collection of items of information, depending on whether we emphasize its structure (logical aspect), its content (set-theoretic aspect), or the relation of the items to real events (factual aspect).
From the practical point of view, an item of information will be defined as a quadruple (attribute, object, value, confidence). "Attribute" refers to a function that attaches a value (or a set of values) to the object whose name figures in the item of information. This value corresponds to a predicate, that is to say, to a subset of the reference domain associated with the attribute. The confidence is an indication of the reliability of the item of information. Clearly the four entities making up the item of information can be composite (several objects, many attributes, n-ary predicate, different degrees of confidence). Moreover, variables can be introduced, especially for the objects, if the item of information involves quantifiers.

In this context we can clearly distinguish the concepts of imprecision and uncertainty: imprecision relates to the content of an item of information (the "value" component of the quadruple) while uncertainty relates to its truth, understood as its conformity to a reality (the "confidence" component of the quadruple).

The uncertainty of an item of information can be assessed by means of qualifiers such as "probable," "possible," "necessary," "plausible," or "credible," to which we shall attempt to give a precise meaning. The modality of being probable has been extensively studied for two centuries. It has two distinct connotations, one of which is physical, tied to statistical experiments, and concerned with the frequency of occurrence of an event. The other is epistemic: here "probable" refers to a subjective judgment. The modalities of possible and necessary go back to Aristotle, who stressed their duality (if an event is necessary, then its contrary is impossible). Oddly, and in contrast with the concept of "probable," the "possible" and the "necessary" are often considered as all-or-nothing categories. But, like "probable," "possible" has two interpretations: physical (as a measure of the material difficulty of performing an action), and epistemic (as a subjective judgment that does not much commit its maker). "Necessary," on the other hand, is a much stronger notion, in either the physical or epistemic sense (subjective necessity amounts to certainty). It is natural to admit degrees of possibility and of necessity, as for probability (a nuance already present in everyday language, where one says "very possible" for example). The connotations of "plausible" and "credible" are specifically epistemic and refer respectively to "possible" and "necessary." Each corresponds to a mode of inference based on a given corpus of knowledge: anything that can be deduced from the corpus is credible; anything that does not contradict the corpus is plausible (inductive aspect). As to "likely," usage seems to give it a sense near to "probable," while "plausible" seems closer to "conceivable."