A NEW METHOD FOR CALCULATION OF LASER-GENERATED ULTRASOUND PULSES

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INTRODUCTION

The generation of acoustic pulses (in solids) by laser pulses has received considerable attention recently (an extensive review has been given by Hutchins[1]). Current applications are to nondestructive evaluation and materials characterization, where it is convenient to have a highly reproducible source requiring no contact with the sample [2-5]. The need to make these applications quantitative requires a theoretical model which: 1) is based on fundamental principles; 2) allows the use of realistic sample and source properties; and 3) is readily usable by the research community without a major computational development effort. Doyle[6] and Schliechert et al.[7] have described approaches which meet the first two criteria, but which are very computation-intensive. We will describe and illustrate a new formulation[8] which meets all three criteria. Numerical calculations will be presented to illustrate the efficacy of this approach, with emphasis on the effects of finite source dimensions and sample surface modification. Comparison with previous point-source results will indicate when the latter may be used. Finally, we show that the small initial displacement "spike" observed in experiments with metal samples, is due to "mode conversion" (thermal-to-longitudinal) at the boundary, rather than to the finite size of the thermal source resulting from thermal diffusion. For the present we limit the discussion to the thermoelastic regime.

The problem of thermally-generated elastic pulses was first considered by Danilovskaya[9], employing the (partially) coupled equations of classical thermoelasticity (CTE) (see Nowacki[10]). Related single-variable (1-D) treatments more specifically directed toward laser-generated ultrasound have been given by White[11] and many others (see Hutchins[1]). The most difficult case to treat theoretically is that of pulse generation in bounded solids in three dimensions, because of the (possible) occurrence of transverse, Rayleigh, and Lamb modes. One-dimensional models will not suffice: important features of the generated waveforms occur only when three-dimensional aspects are considered. Scruby et al. [12, 13] obtained considerable insight from a model invoking equivalent forces to represent a point thermal (surface) source, and Rose [14] has extended that understanding and given it a more rigorous foundation with an analytical development based on a point temperature (surface) source. Experimental measurements (see reviews by Hutchins [1] and Hutchins and Tam [3]) are in good agreement, except for the small, positive "precursor" (mentioned
above), the source of which has not been certain. Doyle[6], Schliechert et al.[7], and Telschow and Conant[15] have attributed this precursor to the finite source dimensions arising from thermal diffusion; however, see below.

THEORETICAL FORMULATION

Quantitative applications of laser-generated pulses would benefit from a rigorous theoretical basis and the flexibility of general sample and source properties and configurations. Very recently a new formulation of pulsed photoacoustic generation (PPG) has been outlined[8]. This formulation utilizes numerical Hankel-Laplace transform inversion, and is based on generalized thermoelasticity (GTE) theory. The most important feature is the (relative) ease of numerical transform inversion, as demonstrated in the related problem of pulsed photothermal deformation[16-18]. The essential new element from GTE (see Chandrasekhariah[19]) is a hyperbolic equation of heat transport with a resulting finite speed of heat propagation (typically somewhat larger than the longitudinal speed). The latter feature gives one confidence that no unphysical behavior is being introduced by GTE, with its associated "infinite" speed of heat propagation, though no appreciable effect on numerical results has been found; it also simplifies certain analytical results.

The medium in which the elastic pulses are generated is assumed to be homogeneous and isotropic. The elastic displacement is obtained through the solution of wave equations for the displacement potentials \( \phi \) and \( \psi \):

\[
\ddot{u} = \nabla \phi + \text{curl} (\dot{\psi} \hat{\theta}),
\]

with

\[
\nabla^2 \phi - \frac{1}{c_1^2} \frac{\partial^2 \phi}{\partial t^2} = \delta T,
\]

\[
(\nabla^2 \frac{1}{r^2}) \psi - \frac{1}{c_2^2} \frac{\partial^2 \psi}{\partial t^2} = 0,
\]

where \( \hat{\theta} \) is the azimuthal unit vector, \( c_1 \) and \( c_2 \) are the longitudinal and shear wave velocities, respectively, \( T \) is the temperature (above ambient, \( T_0 \)), and \( a \) is the thermoelastic coupling constant, \( a = B\beta/\rho c_1^2 \), with \( B \) the bulk modulus, \( \beta \) (the volume) thermal expansion coefficient, and \( \rho \) the density. The (hyperbolic) time-dependent thermal diffusion equation for the temperature is written:

\[
-\kappa \nabla^2 T + \frac{\kappa}{c_T^2} \frac{\partial^2 T}{\partial t^2} + \rho C \frac{\partial T}{\partial t} + B\beta T_0 \frac{\partial \nabla^2 \phi}{\partial t} = S_T(r, t),
\]

with \( c_T \) the thermal velocity, \( \rho C \) the volume heat capacity, \( \kappa \) the thermal conductivity, and \( S_T(r, t) \) the heat source (usually arising from optical absorption). The Laplace and Hankel transform operations are performed; thermal and elastic boundary conditions are imposed to enable the algebraic determination of coefficients for the transform solutions; and the required inverse transforms are obtained, analytically where possible, otherwise numerically (see below). At present we consider only axisymmetric problems, so each potential is represented by a single Bessel function (order 0 for \( \phi \), order 1 for \( \psi \)). The (double-transformed) source term in Eq.(4) has the form

\[
\hat{S}_T = \hat{F}(p)\hat{F}(s)\hat{J}(z),
\]