STRUCTURE AND DYNAMICS OF NONLINEAR
CONVECTIVE STATES IN BINARY FLUID MIXTURES

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Various properties of traveling wave (TW) and stationary overturning convection (SOC)
solutions for ethanol–water parameters by finite-differences numerical solutions of the basic
hydrodynamic field equations subject to realistic horizontal boundary conditions. Bifurcation- and
phase diagrams for TW and SOC solutions are presented. Unstable SOC patterns that decay into a
stable TW or the conductive state can be stabilized by phase pinning lateral boundaries. The
structural changes at the transition TW ↔ SOC are shown. The mean flow, the lateral currents of
heat and concentration, and the particle motion associated with TW are elucidated.

I INTRODUCTION

Convection in horizontal layers of binary fluid mixtures heated from below has
attracted in recent years a great deal of experimental and theoretical activities¹⁻²⁰. Of
particular interest has been the primary pattern forming instability of the motionless
conductive state and the structure, dynamics, and bifurcation behaviour of the
convective states that bifurcate out of the basic conductive state. In a pure fluid the
instability that occurs first on increasing the Rayleigh number

\[ R = \frac{\alpha gd^3}{\kappa \nu} \Delta T \]  \hspace{1cm} (1.1)

beyond the threshold \( R_c^0 = 1707.8 \) is a stationary one. The conductive state loses
stability in a forwards bifurcation to a stable pattern of stationary overturning
convection (SOC) rolls. However, in a mixture the primary instability is either
oscillatory or stationary depending on the separation ratio

\[ \psi = -\beta T_0 - k_T \]  \hspace{1cm} (1.2)

Here \( d \) is the vertical thickness of the layer, \( \kappa \) the thermal diffusivity, \( \nu \) the kinematic
viscosity, \( T_0 \) the mean temperature, \( \alpha = - (1/\rho) \partial \rho(T,p,C)/\partial T \) the thermal expansion
coefficient, \( \beta = - (1/\rho) \partial \rho(T,p,C)/\partial C \) the solutal expansion coefficient, and \( k_T \) the
thermodiffusion ratio of the fluid mixture. Furthermore, \( g \) is the gravitational constant and \( \Delta T \) the vertical temperature difference across the fluid layer.

In Fig. 1 we show schematically the reduced Rayleigh numbers

\[
    r = \frac{R}{R_c^0}
\]

(1.3)

Fig. 1. Schematic bifurcation lines \( r_{osc} \) and \( r_{stat} \) of oscillatory and stationary convection out of the conductive state. Plotting the vertical convective heat current in an \( N \) vs \( r \) diagram \( TW \) (SOC) convection bifurcates forwards \( \partial N / \partial r > 0 \) out of the conductive state across the full part of the \( r_{osc}(r_{stat}) \) line and backwards \( \partial N / \partial r < 0 \) across the dashed part. At the tricritical values \( \psi^t \) the initial slope diverges: \( N - 1 \propto (r - r_t)^{1/2} \). Bifurcating SW solutions are not considered here. For \( \psi = 0 \) one has the behaviour of a pure fluid. The diagram is not to scale: For room temperature ethanol—water mixtures \( \psi^{t}_{SOC} = \mathcal{O} (-10^{-6}), \psi^{t}_{TW} = \mathcal{O} (-10^{-4}) \).

where convective solutions branch off the conductive state. From experiments\(^{1-5}\), linear stability analysis\(^{6, 7}\) of the conductive state, few-mode Galerkin approximations\(^{8-11}\), and weakly nonlinear perturbation analysis\(^{12}\) of the bifurcating convective states\(^{13, 14}\) has emerged the following picture: (1) In the \( r-\psi \) plane the stability range of the conductive state is bounded by an oscillatory bifurcation threshold, \( r_{osc} \), and a stationary threshold, \( r_{stat} \). (2) On the stationary branch, \( r_{stat} \) of the stability boundary there is at \( \psi^{t}_{SOC} \) a tricritical point in the sense of Landau's meanfield classification of phase transitions. There the vertical convective heat current flowing through the layer grows with divergent initial slope: \( N - 1 \propto (r - r_t^{soc})^{1/2} \) with \( N \) being the Nusselt number. Above (below) \( \psi^{t}_{SOC} \) there is a forwards nonhysteretic (backwards hysteretic) transition at \( r_{stat}^{soc} \) and the convective