INTRODUCTION

Many optical and transport experiments on layered semiconductor structures involve the continuous energy spectrum. One important example are the quantum wells in a homogeneous electric field; the overall spectrum becomes continuous, bound states transform into resonances. Another example are the tunnelling structures (single barrier or double barrier). Very often the analysis of the continuous spectrum is performed in order to determine the dynamic characteristics of the structures, e.g. lifetimes of quasi-bound states\(^1\),\(^2\), high-frequency properties of tunneling diodes\(^3\),\(^4\) etc. Resonances are often treated with perturbative methods, i.e. the interaction between the discrete state and the continuum is assumed to be weak.

In the present paper we would like to demonstrate, that the direct determination of the density of states (DOS) is a simple and general method for characterizing the continuous spectrum. Apart from the resonances that can be broad (strongly coupled to the continuum) the study of the DOS reveals many interesting structures in the continuum, directly related to the behaviour of the wavefunctions. Furthermore we show that DOS is, in same cases, a better characteristic of structures in the continuous spectrum than transmission. First we briefly describe the method for determining the DOS, then we apply it to three important systems: tunneling structures, single quantum wells and the quantum wells in a uniform electric field.

METHOD

We put the considered structure in a big box so that the spectrum is dense but discrete. Suppose the condition for the energy levels \( E_n \) in an empty box is:

\[
D(E_n^0) = 0 ,
\]

and for a box with the structure:

\[
D(E_n) = 0 .
\]
The DOS is given by the spacing between the levels:

\[ \rho_0(E_n) = \frac{1}{\Delta_n^0}, \]

\[ \rho(E_n) = \frac{1}{\Delta_n}, \]

where \( \Delta_n^0 = E_{n+1}^0 - E_n^0 \), \( \Delta_n = E_{n+1} - E_n \). Both of these densities increase with the box size. However, the difference between them, which characterizes the considered structure, stays finite even for an infinite box. Thus we can write:

\[ \Delta_n = \Delta_n^0 + x_n \]

(5)

with \( |x_n| \ll \Delta_n^0 \). Therefore the change of the density of states \( \Delta \rho \), from Eqs. (3)-(4), becomes:

\[ \Delta \rho = \frac{1}{\Delta_n^0} - \frac{1}{\Delta_n} \equiv -\frac{x_n}{(\Delta_n^0)^2} \]

(6)

The shift \( x_n \) can be obtained by expanding \( D(E) \) around \( E_{n+1}^0 \), and we finally obtain:

\[ \Delta \rho \equiv \frac{1}{(\Delta_n^0)^2} \frac{D(E_n^0+\Delta_n^0)}{D'(E_n^0+\Delta_n^0)} \]

(7)

where \( D' \) indicates the first derivative of \( D \).

Thus, if we know the eigenvalue condition (2) we can easily determine \( \Delta \rho \). The spacing \( \Delta_n^0 \) in an empty box is often known exactly (e.g. \( \Delta_n^0 = (\hbar/2L)^2 \sqrt{2m^*} \) for a particle of mass \( m^* \) in a flat box of size \( L \)) but it can always be determined from Eq. (1).

APPLICATION TO TUNNELING STRUCTURES

This part of the work has already been published\(^5\), therefore we only stress the main conclusions. The DOS seems to be a better characteristic of resonances than transmission, which is usually applied in this case. For narrow resonances both methods give the same positions and widths (Fig. (1a)), for broad structures the transmission is more obscure (Fig. (1b)).

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Fig. (1) Comparison between DOS and transmission for a double barrier structure. The barriers are 50 Å wide and 200 meV high, and the well is 100 Å wide; the effective mass \( m^* \) is \( m^* = 0.067 m_0 \) (electrons for GaAs)