The microemulsion model was proposed by B. Widom for the description of the microfilm structure in the "oil-water" mixture with some values of the mixture parameters. The model is a classical 3-D spin lattice model. The spin takes the values \( \pm 1 \) and the interaction is described by the Hamiltonian:

\[
H(I^2) = -I \sum_{\text{dist}(x, y) = 1} \phi(x) \phi(y) - J \sum_{\text{dist}(x, y) = \sqrt{2}} \phi(x) \phi(y) - K \sum_{\text{dist}(x, y) = 2} \phi(x) \phi(y),
\]

where \( \phi(Z^3) \) is a configuration on \( Z^3 \), \( x, y \in Z^3, J=2K \). This Hamiltonian includes the interaction of nearest neighbours in the first sum, the interaction of diagonal neighbours in the second sum and the interaction of next nearest neighbours in the third sum.

We shall consider a more general model with arbitrary \( J, K \) and \( D>0 \). The model with \( K<0 \) is transformed into the previous one if the signs of all spins with even coordinates are changed.

Usually for models of this type a low-temperature phase diagram is a small perturbation of a ground states diagram. It is well known from the Pirogov-Sinai theory that this situation arises in particular in models with the finite number of the ground states satisfying the Peierls stability condition: the energy jump on the boundary between two ground states is proportional to the boundary area.

Therefore it is natural to begin the investigation of the Hamiltonian (1) by studying its ground states. With this purpose we can rewrite the initial Hamiltonian as the following double sum:

\[
\sum_{x \in Z^3} \phi(x) \sum_{y \in N(x)} \left[ -0.5J \sum_{\text{dist}(x, y) = 1} \phi(y) - 0.5J \sum_{\text{dist}(x, y) = \sqrt{2}} \phi(y) - K \sum_{\text{dist}(x, y) = 2} \phi(y) \right]
\]

where \( N(x) \) is the set of the nearest neighbours of the lattice site \( x \). The minimum of each item of the external sum is reached on some family \( \Phi(I,J,K) \) of the configurations in the volume \( N(x) \). Due to the symmetry of the Hamiltonian the family \( \Phi(I,J,K) \) does not depend on the site \( x \) and is invariant with respect to the permutation of the coordinate axes and to changing of all spin signs.

Obviously if there exists a periodic configuration
\[ \phi(Z^3) : \phi(N(x)) = \Phi(I,J,K) \text{ for all } x \] (3)

then it is a ground state. Moreover in this case all periodic ground states are the configurations of type (3) and are uniquely defined by the family \( \Phi(I,J,K) \).

The diagram of the families \( \Phi(I,J,K) \) is shown in Fig. 1. It contains six domains with different \( \Phi(I,J,K) \). Only one configuration is drawn for each \( \Phi(I,J,K) \) meaning that all other configurations can be obtained from this one after the permutation of the axes and/or after changing of all spins sign.

It can be verified that in the interior of each domain \( Q_1 - Q_6 \) the corresponding family \( \Phi(I,J,K) \) generates the finite number of the ground states satisfying the Peierls condition.

On the domains boundaries the family \( \Phi(I,J,K) \) is the union of the families from the contiguous domains. This increasing of the \( \Phi(I,J,K) \) leads to a violation of the Peierls condition. Hence the phase diagram analysis becomes much more complicated in the neighbourhood of this boundaries.

We can construct the low-temperature phase diagram of the model only in those parts of parameter space where all periodic ground states are layered configurations. Recall that a configuration is called layered if it takes the constant value on any lattice plane orthogonal to one coordinate axis. For such configuration \( \varphi \) a sequence of \( l \) adjacent planes is called the layer of thickness \( l \) if \( \varphi \) takes the same constant value on these planes and the opposite value on the planes which are nearest to these ones. The finite sequence \( < l_1, \ldots, l_k > \) of integer positive numbers defines the class of layered periodic configurations. The class is generated by the sequence of adjoining layers of thicknesses \( l_1, l_2, \ldots, l_k \). We use a special notation \( < \infty > \) for the class containing two ferromagnetic configurations.

Inside the domain \( Q_1 \) ground states are the ferromagnetic configurations, inside the domain \( Q_2 \) - configurations \( <1> \), inside the domain \( Q_3 \) - configurations \( <2> \). On the interior of the \( Q_1 \) and \( Q_2 \) common boundary there exists the infinite set of ground states consisting of any periodic layered configurations containing only the layers of thickness 1 or 2. On the interior of the \( Q_1 \) and \( Q_3 \) common boundary an arbitrary layered configuration is a ground state if it does not contain layers of thickness 1. In the triple point \( I=4J, K=0 \) any layered configuration is a ground state. On the interior of the \( Q_1 \) and \( Q_2 \) common boundary only configuration \( <\infty> \) and \( <1> \) are ground states and Peierls condition is satisfied.

The stability analysis of the ground states described has shown that the low-temperature phase diagram is not a small perturbation of the ground states diagram. The diagram contains only a finite number of the phases among which the first order phase transitions are occured. The exact picture of phase transitions is described by the following theorem.

**Theorem.** For each fixed \( I \) and large enough inverse temperature \( \beta \) \((\beta > 1)\):

- in a small neighbourhood of the triple point \( J=I/4, K=0 \) there exists a full phase diagram for the phases \( <1>, <2>, <\infty> \) (see Fig. 2.a);
- in a small neighbourhood of the interval \( -I/2 < K \Phi^{-1} \text{ on the line } I+4J+2K=0 \) finite number of phases from the set: \( <2>, <3>, <4>, \infty > \) survive (see Fig. 2.b);
- in a small neighbourhood of the interval \( -I/6 < K \Phi^{-1} \text{ on the line } I+4J-2K=0 \) finite number of phases from the set: \( <1>, <2>, <2.1> \) survive (see Fig. 2.c).

It is necessary to take into account the entropy effects to understand why at non zero temperature only a finite number of phases survive.