TRANSITIONS TO MORE COMPLEX PATTERNS IN THERMAL CONVECTION

F.H. Busse and R.M. Clever
Institute of Physics, University of Bayreuth, 8580
Bayreuth, FRG, and Institute of Geophysics and Planetary
Physics, UCLA, Los Angeles, USA

SUMMARY

The instabilities of convection rolls in a fluid layer heated from
below are reviewed and results of recent computations on
three-dimensional knot convection flow and on travelling wave convection
are reported. Periodic boundaries in the horizontal directions and rigid,
thermally well conducting boundaries at top and bottom have been assumed.
The analysis of the stability of the three-dimensional convection
patterns indicates transitions to a variety of time-dependent forms of
convection which in part can be related to experimental observations.
Further experimental work on low Prandtl number fluids appears to be
desirable for the study of convection in the form of standing waves.

1. INTRODUCTION

Thermal convection in an extended fluid layer heated from below is
unique among hydrodynamic instabilities in that a maximum number of
symmetries is available and in that more transitions from simple to more
complex patterns of flow can be followed than in other fluid systems.
Moreover, there are a number of external parameters available which
either reduce the symmetry such as an imposed horizontal magnetic field
or do not change the symmetry of the fluid layer, but provide a useful
control parameter and introduce new degrees of freedom such as a vertical
magnetic field. In this review we give a brief survey of patterns of
nonlinear solutions and of their instabilities using symmetry
considerations as a guide. In section 2 the instabilities of convection
rolls will be considered, in section 3 three-dimensional knot-solutions
will be discussed and in section 4 some properties of travelling wave
convection in low Prandtl fluid will be outlined. The paper closes with
some concluding remarks on open problems and work in progress.

2. INSTABILITIES OF CONVECTION ROLLS

We consider steady two-dimensional solutions of the basic equations
describing convection rolls in a horizontal layer heated from below. The
Table 1. Properties of Instabilities of Convection Rolls in a Layer with Rigid Boundaries. The quantity $2\pi/\Omega$ is a measure of the circulation time in rolls.

<table>
<thead>
<tr>
<th>Instability</th>
<th>Symmetry Class</th>
<th>b</th>
<th>d</th>
<th>$\sigma_i$</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>cross-roll</td>
<td>OC</td>
<td>$&gt;\alpha_c$</td>
<td>0</td>
<td>0</td>
<td>CR</td>
</tr>
<tr>
<td>knot</td>
<td>OC</td>
<td>$&lt;\alpha_c$</td>
<td>0</td>
<td>0</td>
<td>KN</td>
</tr>
<tr>
<td>dual blob</td>
<td>OC</td>
<td>$&lt;\alpha_c$</td>
<td>0</td>
<td>$2\Omega$</td>
<td>DB</td>
</tr>
<tr>
<td>zig-zag</td>
<td>ES</td>
<td>$&lt;\alpha_c$</td>
<td>0</td>
<td>0</td>
<td>ZZ</td>
</tr>
<tr>
<td>oscillatory</td>
<td>ES</td>
<td>$&lt;\alpha_c$</td>
<td>0</td>
<td>$\Omega$</td>
<td>OS</td>
</tr>
<tr>
<td>single-blob</td>
<td>EC</td>
<td>$&gt;\alpha_c$</td>
<td>0</td>
<td>$\Omega$</td>
<td>SB</td>
</tr>
<tr>
<td>skewed-varicose</td>
<td>E</td>
<td>$&lt;\alpha_c$</td>
<td>$&lt;\alpha_c$</td>
<td>0</td>
<td>SV</td>
</tr>
<tr>
<td>Eckhaus</td>
<td>E</td>
<td>0</td>
<td>$&lt;\alpha_c$</td>
<td>0</td>
<td>EC</td>
</tr>
</tbody>
</table>

deviation $\theta$ of the temperature from the temperature distribution of the state of pure conduction can be used as the representative dependent variable of the convection flow. Using a Cartesian system of coordinates with the $z$-coordinate in the direction opposite to gravity and the $x$-coordinate in the direction of the axis of the rolls we can write the solution for $\theta$ in the form

$$\theta = \sum_{m,n} b_{mn} \cos\omega y \sin\pi(z+\frac{1}{2})$$  \hspace{1cm} (2.1)

Anticipating that the solutions of interest possess a vertical plane of symmetry, we have located the origin on the intersection between such a plane and the median plane of the layer. Assuming a Boussinesq fluid and symmetric boundary conditions we find that the convection rolls bifurcating from the static solution of the problem at the critical Rayleigh number $R_c$ satisfy the additional symmetry property

$$\theta(y,z) = -\theta(\frac{\pi}{\alpha}y,-z)$$  \hspace{1cm} (2.2)

According to this property all coefficients $b_{mn}$ with odd $m+n$ vanish in the representation (2.1). Solutions of the form (2.1) can be obtained for a wide range of the $R$-$\alpha$-$P$ parameter space where $R$ is the Rayleigh number and $P$ is the Prandtl number. We refer to Busse (1967), Clever and Busse (1974, 1978), Busse and Clever (1979) and Bolton, Busse and Clever (1986).

General three-dimensional infinitesimal disturbances of the steady solution given by (2.1) can be written in the form

$$\theta = \sum_{m,n} \tilde{b}_{mn} \exp\{im\alpha y+idy+ibx+\sigma t\} \sin\pi(z+\frac{1}{2})$$  \hspace{1cm} (2.3)

where Floquet's theorem has been used. Because of property (2.2) the general disturbances of the form (2.3) separate into two classes. Those of class $E$ have vanishing coefficients $\tilde{b}_{mn}$ for odd $n+m$, while the coefficients of class $0$ vanish for even $n+m$. It turns out that the real