PRESSURE DEPENDENCE OF X-RAY YIELDS
IN PROTONIUM

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In view of the interest in studying the pp system in gas
at low to moderate pressures, it has seemed worthwhile to extend
previous calculations of the atomic cascade to provide the best
possible estimates of the yields of K- and L-x rays in protonium
as a function of target pressure. The processes involved -
Stark mixing, annihilation, chemical (or Coulomb) and Auger
deeexcitation, as well as radiative transitions, have been
described in Refs. 1 and 2. In comparison to previous work the
following changes have been made:

1: The shift and width of the 1s state have been taken from
the most recent calculations by Richard and Sainio. The calculation
by Kaufmann, based on a boundary condition model, gives
essentially the same result, namely $\Gamma_{1s}=700 \text{ eV}, \Delta E_{1s}=1000 \text{ eV}$ (spin
averaged). In most of the present work, the annihilation width
of the 2p state was taken to be 30 meV, or 75 times the radiative
width, also in accordance with the calculations of Refs. 4 and 5.

2: The role of Coulomb deexcitation, as proposed by Bracci
and Fiorentini has been studied. For this purpose, the rate for
chemical deexcitation

$$\Gamma_{if}^{(\text{chem})} = \frac{1}{2} N \nu \pi a_{n_i}^2 , \quad \Delta E > 4.7 \text{ eV}$$

was replaced by an approximation to the rates calculated in
Ref. 6, namely

$$\Gamma_{if}^{(\text{Coul})} = \frac{N \nu}{20} \pi a_{n_i}^2 \left(1 + 27.2 \frac{M_p}{T n_i^2}\right) , n_f=n_i-1$$
where \( M = \frac{m_{\text{red}}}{m_{\text{e}}} \) and \( T \) is the atom's kinetic energy in eV. This formula gave approximately the same results for muonic hydrogen as the fit given in Eq. (49) of Ref. 6.

3: Initial distributions in \( n \) and \( l \) were taken from the work of Cohen, Martin and Wadt\(^7\). However, the results for the x-ray yields differed insignificantly from those calculated using a statistical distribution starting at \( n=31 \), even at very low pressures.

4: The possibility of annihilation from \( d \)-states was also taken into account. According to Ref. 4, the annihilation width of the \( 3d \) state is about 0.4 \( \mu \text{eV} \). The \( n \)-dependence is given by

\[
\Gamma_{nd} = \frac{2187}{40} \left( n^4 - 5n^2 + 4\right)/n^7 \Gamma_{3d}.
\]

At very low pressures (<50 torr) this resulted in about 1% of the \( pp \) systems annihilating from a \( d \)-state. At higher pressures, Stark mixing suppressed the annihilation from \( d \)-states. The Day, Snow, Sucher effect\(^8\) is less operative for the case of \( p \)-states due to the very large annihilation width. According to Ref. 9, \( \Gamma_{2p}(\text{ann}) > 10 \Gamma_{2p}(\text{rad}) \), while theory\(^4,5\) predicts an even larger ratio of annihilation to radiative widths.

In the present work, we kept the Stark mixing rate multiplier and kinetic energy fixed at \( k_{STK} = 2 \) and \( T = 1 \text{ eV} \), respectively. At low pressures, the x-ray yields are not affected much by this assumption. The sensitivity of results to these parameters has been discussed in Ref. 1. The values chosen here gave reasonable agreement with the present rather limited experimental information\(^9\). A similar calculation for muonic hydrogen, using the same parameters gave values of \( K_a/K_{\text{tot}} \) in good agreement with experiment\(^10,11\) for several pressures. Also, this ratio is not very sensitive to these parameters. It is also not sensitive to the initial distributions or to the details of the deexcitation mechanism in the early stages of the cascade, in the sense that all of these can be varied within reasonable limits without affecting the value of \( K_a/K_{\text{tot}} \) by more than a few per cent.

Our results for the \( pp \) system are displayed in Figs. 1-4. In each case we show results calculated using either chemical or Coulomb deexcitation for two different values of \( \Gamma_{2p}(\text{ann}) \), namely 30 meV, as predicted by theory\(^4,5\) and 4 meV, the lower limit given in Ref. 9. Fig. 1 shows \( K_a/K_{\text{tot}} \), which decreases from about 0.9 at very low pressures to less than 0.05 in liquid hydrogen. The range of values displayed in the curves indicates the rather weak sensitivity of this result to assumptions about the deexcitation mechanism early in the cascade, velocity of the \( pp \) atom, exact value of Stark mixing rates, etc. Results calculated using other values of the adjustable parameters are similar. In liquid,