About 2500 years ago, a Greek named Pythagoras walked by Homer's fabled wine-dark sea, until he came to know what would inform his whole life. Arithmetic, with grand contempt for the slippery pebbles' uncertain support, boldly vaulted from the earth—and geometry was drawn out of the stars. Mathematics sprang from this marriage.

In childhood, mathematics was nurtured by that early spirit rising from the shores of the Mediterranean, exhorting Greeks to walk like giants, to wrest secrets from the gods. The history of mathematics ever since has been bound up with the workings of that spirit. What was it like, back then? Whose was the voice that howled above the seashore, as Pythagoras lengthened his stride? What moved the train—Eudoxus to Archimedes—that followed Pythagoras down the shore?

Let us study the workings of that train, to come to know its spirit. We ourselves move in it. We are still transported by a caravan of Greeks—who felt mathematics spring from head to shoulder—upon whose shoulders we too stand.

The purpose of the present chapter is to remind ourselves of history. Some readers may have the urge instead to jump flat-footed into an attack upon the problem that arose in Chapter 1: the problem of how the highest and lowest points on a curve can be easily found. Those readers may jump to Chapter 3, but they are warned that flat-footed jumps are awkward without a good foundation from which to leap. Studying history helps to build foundations.

§1. The Philosophy of Pythagoras

Real mathematics begins with Pythagoras (ca. 569–500 B.C.), although small steps were taken earlier by the Sumerians, Babylonians, and Egyptians. Some would say that Thales, who taught Pythagoras, deserves as much

W. M. Priestley, *Calculus: An Historical Approach*
© Springer-Verlag New York, Inc. 1979
credit; but Thales, hailed widely as the “father of philosophy”, surely enjoys honor enough already. Why the spark of mathematics should suddenly glow so brightly (and why the flame would die with the coming of the Romans) is still a mystery.

Geometry became increasingly the dominant theme in Greek mathematics, and Pythagoras set the style. Yet Pythagoras was at first more attracted to arithmetic. He and his followers, the Pythagoreans, founded a small society that virtually worshipped numbers. One short sentence is all it takes to sum up the philosophy of Pythagoras:

All is number!

Pythagoras, it is said, invented the word philosophy, which literally means “love of wisdom”. He sought wisdom by studying numbers. Number, to the Pythagoreans, referred to the ideas they pictured by the sequence

\[
\ldots \quad \cdot \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (\text{etc.})
\]

Today we call these numbers positive integers. Exactly what the Pythagoreans meant by asserting that all is number is not entirely clear. At the least they meant that numbers are connected with many things that, at first, seem totally unrelated to numbers. For example, the musical tones produced by plucked strings seem at first to have nothing at all to do with numbers. Yet it was Pythagoras himself, so legend has it, who discovered what we now call thirds, fifths, octaves, etc., because of the numbers that are naturally associated with their relative pitches. Elementary facts about music are such common knowledge today that we surely underrate their significance. In the sixth century B.C., their discovery must have been astonishing. Imagine! Numbers have something to do with music!

“Perhaps numbers have something to do with everything!” thought Pythagoras. “Perhaps everything is number….” At least this offered a viable alternative to Thales’ philosophy. Thales thought everything was water.

The Pythagoreans bequeathed much to Western culture. Who has not heard of the Pythagorean theorem?

The Pythagorean Theorem. In a right triangle, the square built on the hypotenuse has the same area as the combined area of the squares on the other two sides.