STOCHASTIC QUANTIZATION OF GAUGE FIELDS

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SUMMARY. Stochastic quantization of gauge fields is reviewed, with particular attention to the mechanism by which stochastic gauge-fixing resolves the Gribov ambiguity, and the light which this casts on the geometry of gauge orbits. Some geometric properties of gauge orbits are derived, and the difficulty which they present for the Faddeev-Popov method is reviewed. Stochastic quantization is introduced, and it is shown how the Euclidean probability distribution may be determined by a diffusion process with a drift force $K=-\nabla S$, where $S=S(A)$ is the classical Yang-Mills action. The equivalence of the diffusion and Langevin equations is demonstrated. Stochastic gauge-fixing is explained, and it is demonstrated that an additional drift force, called the gauge-fixing force, may be introduced which does not affect expectation values of gauge-invariant quantities because it is everywhere tangent to the gauge orbits. The gauge-fixing force is non-conservative and thus cannot be accommodated in an action formalism. Properties of the gauge-fixing force are explained and it is shown how, by a mechanism of "instability stabilizes", it concentrates the probability near the interior $\Omega$ of the first Gribov horizon. As a new result, the diffusion equation is solved for large values of the gauge-fixing force, the solution being a

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modified Faddeev-Popov formula, whereby the integration extends only over $\Omega$. Another new result is the \( n \) or loop expansion of the effective action for stochastic gauge-fixing, and it is shown that all primitive divergences arise in the iterative calculation of an effective drift force $K_{\text{eff}}$.

1. INTRODUCTION

The method of stochastic quantization was invented by Parisi and Wu.\(^1\) It may be used to advantage in non-singular systems.\(^2\) Its main attraction however would appear to be the new possibilities which it offers for quantizing systems with constraints arising from symmetries. This is at present a topic of active research in the quantization of the gravitational field\(^3\) and it may prove to be of value for quantizing other systems such as strings. In the present lectures we shall deal only with its application to gauge fields. Current literature on this subject includes, to mention only some articles, a study of the equivalence of stochastic perturbation theory to conventional Feynman diagrams with Faddeev-Popov ghosts\(^4\), proposals to use the fifth time as a gauge-invariant regulator\(^5,6\), functional integral representations of the solution\(^7\), studies of the dependence of gauge-dependent Green’s functions on the initial conditions in the fifth time\(^8\), and numerical simulation of the underlying stochastic process\(^9\).

The basic idea of stochastic quantization is that the Euclidean probability distribution is determined by a diffusion process with a drift force, $K$. In the original formulation of Parisi and Wu\(^1\), the drift force is given by $K = -\nabla S$, where $S$ is the classical Yang-Mills action. With stochastic gauge-fixing\(^10\), this force is modified by the addition of a force that does not affect the expectation values of gauge-invariant observables because it is tangent to the gauge orbits. This force is of necessity non-conservative, and thus cannot be accommodated in an action formalism, but it has the restoring property required to maintain an equilibrium, or time-independent, probability distribution. The analog in quantum gravity would be to introduce a force tangent to the orbits under the group of diffeomorphisms.