ANALYSIS OF THE PRESSURIZING GAS REQUIREMENTS FOR AN EVAPORATED PROPELLANT PRESSURIZATION SYSTEM*

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Introduction

This paper describes the results of an analytical and experimental program for studying the expulsion of liquid from a missile propellant tank by means of an evaporated propellant. The analytical study has been directed toward obtaining a method for accurately determining the pressurizing gas requirements for expulsion. The test program was conducted to determine the required empirical coefficients for the analysis, to provide a means for checking the analysis, and to permit detailed studies of the phenomena taking place as a result of the dynamic and thermodynamic conditions imposed on the system. Although several propellants are being investigated in this study, only the results of the liquid-hydrogen program are presented in this paper.

Analysis

In the prediction of missile propellant tank pressurization requirements, a computer solution is required due to the complex nature of the governing equations. To satisfy this need, an IBM 7090 computer program was developed for application to a booster or upper-stage propellant tank with or without expulsion and with pressurization by a condensable or a noncondensable gas. This program involves the numerical solution of a large system of differential equations which have been written to describe the thermal behavior of various elements in and on the idealized propellant tank shown in Fig. 1. These equations are simply various statements of the first law of thermodynamics and generally become evident when the basic assumption involved is stated. Since they are bulky and generally complicated, they are not presented here but can be found in another paper [1]. Instead, some attention will be devoted to the main assumption that the pressurizing gas is completely thermally mixed.

To determine whether this assumption is justified, an analysis along the lines of Clark et al. [2] was made, except that the pressurizing gas was assumed to be completely mixed and to be compressible instead of nonmixed and incompressible. The analytical model used is a simplified form of that shown in Fig. 1. The following situation is analyzed.

At a constant pressure and inlet temperature, a completely mixed perfect gas fills a cylindrical container having insulated ends and walls with a constant heat capacity, zero axial conductivity, and infinite radial conductivity. The gas expels a liquid at a uniform rate from the initially filled container. There is no heat or mass transfer at the liquid—gas interface; the only heat transfer between the cylindrical walls and the gas is convection, utilizing a constant film co-

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efficient. For generality the container is also exposed to both external convection and to a constant energy source.

The following equations can be written:

\[
C_m \frac{dW_m}{d\theta} - \pi Dh \int_0^{\nu \theta} (T_m - T_w) \, dx = \frac{d}{d\theta} (W_m C_m T_m)
\]  

\[
\frac{\partial T_w}{\partial \theta} = \frac{h}{(tpC)_w} (T_m - T_w) + \frac{K_1}{(tpC)_w} + \frac{U(T_e - T_w)}{(tpC)_w}
\]

with the boundary conditions

\[
T_m(0) = T_{m_0} \quad T_w(\theta, X \geq V\theta) = T_L
\]

Equation (2) can be rewritten as an ordinary differential equation for a general wall element (located at \( X \) for \( X \leq V\theta \) by the substitution \( \theta' = \theta - X/V \). The resulting expression together with (1) can be integrated implicitly to give

\[
T_m = \frac{T_{m_0}}{1 + \left( \frac{4R_m - h}{P_m C_m DV \theta} \right) \int_0^{\nu \theta} (T_m - T_w) \, dx \, d\theta}
\]

\[
T_w = \frac{UT_e + K_1}{h + U} \left\{ \left( 1 - \exp \left[ - \frac{h + U}{(tpC)_w} \left( \theta - \frac{X}{V} \right) \right] \right) + T_L \exp \left[ - \frac{h + U}{(tpC)_w} \left( \theta - \frac{X}{V} \right) \right] 
\]

\[
+ \frac{h}{(tpC)_w} \left[ \exp \left[ - \frac{h + U}{(tpC)_w} \theta \right] \int_{\theta = X/V}^{\theta} \exp \left[ \left( \frac{h + U)\theta}{(tpC)_w} \right] T_m \, d\theta \right. \right. \}
\]