All Is Not Well with Classical Mechanics

It was mentioned in the Prelude that as we keep expanding our domain of observations we must constantly check to see if the existing laws of physics continue to explain the new phenomena, and that, if they do not, we must try to find new laws that do. In this chapter you will get acquainted with experiments that betray the inadequacy of the classical scheme. The experiments to be described were never performed exactly as described here, but they contain the essential features of the actual experiments that were performed (in the first quarter of this century) with none of their inessential complications.

3.1. Particles and Waves in Classical Physics

There exist in classical physics two distinct entities: particles and waves. We have studied the particles in some detail in the last chapter and may summarize their essential features as follows. Particles are localized bundles of energy and momentum. They are described at any instant by the state parameters \( q \) and \( \dot{q} \) (or \( q \) and \( p \)). These parameters evolve in time according to some equations of motion. Given the initial values \( q(t_i) \) and \( \dot{q}(t_i) \) at time \( t_i \), the trajectory \( q(t) \) may be deduced for all future times from the equations of motion. A wave, in contrast, is a disturbance spread over space. It is described by a wave function \( \psi(r, t) \) which characterizes the disturbance at the point \( r \) at time \( t \).

In the case of sound waves, \( \psi \) is the excess air pressure above the normal, while in the case of electromagnetic waves, \( \psi \) can be any component of the electric field vector \( E \). The analogs of \( q \) and \( \dot{q} \) for a wave are \( \psi \) and \( \dot{\psi} \) at each point \( r \), assuming \( \psi \) obeys a second-order wave equation in time, such as

\[
\nabla^2 \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}
\]
which describes waves propagating at the speed of light, c. Given \( \psi(r, 0) \) and \( \psi(r, 0) \) one can get the wave function \( \psi(r, t) \) for all future times by solving the wave equation.

Of special interest to us are waves that are periodic in space and time, called plane waves. In one dimension, the plane wave may be written as

\[
\psi(x, t) = A \exp \left[ i \left( \frac{2\pi}{\lambda} x - \frac{2\pi}{T} t \right) \right] = A \exp[i\phi]
\]

(3.1.1)

At some given time \( t \), the wave is periodic in space with a period \( \lambda \), called its wavelength, and likewise at a given point \( x \), it is periodic in time, repeating itself every \( T \) seconds, \( T \) being called the time period. We will often use, instead of \( \lambda \) and \( T \), the related quantities \( k = 2\pi/\lambda \) called the wave number and \( \omega = 2\pi/T \) called the (angular) frequency. In terms of the phase \( \phi \) in Eq. (3.1.1), \( k \) measures the phase change per unit length at any fixed time \( t \), while \( \omega \) measures the phase change per unit time at any fixed point \( x \). This wave travels at a speed \( v = \omega/k \). To check this claim, note that if we start out at a point where \( \phi = 0 \) and move along \( x \) at a rate \( x = (\omega/k)t \), \( \phi \) remains zero. The overall scale \( A \) up front is called the amplitude. For any wave, the intensity is defined to be \( I = |\psi|^2 \). For a plane wave this is a constant equal to \( |A|^2 \). If \( \psi \) describes an electromagnetic wave, the intensity is a measure of the energy and momentum carried by the wave. [Since the electromagnetic field is real, only the real part of \( \psi \) describes it. However, time averages of the energy and momentum flow are still proportional to the intensity (as defined above) in the case of plane waves.]

Plane waves in three dimension are written as

\[
\psi(r, t) = A e^{i(kr - \omega t)}, \quad \omega = |k|v
\]

(3.1.2)

where each component \( k_i \) gives the phase changes per unit length along the \( i \)th axis. One calls \( k \) the wave vector.\( \dagger \)

3.2. An Experiment with Waves and Particles (Classical)

Waves exhibit a phenomenon called interference, which is peculiar to them and is not exhibited by particles described by classical mechanics. This phenomenon is illustrated by the following experiment (Fig. 3.1a). Let a wave \( \psi = A e^{i(kr - \omega t)} \) be

\( \dagger \) Unfortunately we also use \( k \) to denote the unit vector along the \( z \) axis. It should be clear from the context what it stands for.