COMPLEX BEHAVIOUR OF SYSTEMS DUE TO SEMI-STABLE ATTRACTORS: ATTRACTORS THAT HAVE BEEN DESTABILIZED BUT WHICH STILL TEMPORARILY DOMINATE THE DYNAMICS OF A SYSTEM

Byers, R.E., R.I.C. Hansell, and N. Madras

1. Department of Zoology, University of Toronto, 25 Harbord St., Toronto, Ontario, M5S 1A1

2. Department of Mathematics, York University, 4700 Keele St., Downsview, Ontario, M3J 1P3

Abstract

Semi-stable attractors are provisionally defined as invariant sets around which volumes are contracting. Three examples are referred to: one is extensively documented by Grebogi, et al. (1986), a second is generated by an age-structured model, and the last is seen in the logistic model. The first example is described by Grebogi as a chaotic transient. However, this transient lasts in excess of 80,000 iterations, which, for real ecological systems in which each iteration represents a year, is longer than many systems can survive. The second looks very much like a saddle point, and indeed, saddle points may be semi-stable attractors, but only if the absolute value of the determinant of the Jacobian at the saddle point is less than 1. In the last example, the trajectories don't stay near the semi-stable attractor very long, but they generate interesting dynamics when 'r' is permitted to vary, and illustrate the relation between semi-stable attractors and the stable attractors they're derived from.

Introduction

Simple models, like the discrete logistic model, show a wide variety of stable behaviours, ranging from simple equilibria to chaos. In such simple models, transient behaviour disappears very quickly. However, in complicated model systems, some transient behaviours may take a considerable period of time to disappear. For example, Grebogi, et al (1986, Battelino, et al. 1988) describe 'chaotic transients' that 'seem to occur often' and take 'extraordinarily long times' to converge to the relevant periodic attractor (see also Carroll et al 1987, 1988). The extraordinarily long times Grebogi, et al, refer to are between 20,000 and 80,000 iterations (see e.g. figure 7 in Battelino et al, 1988). If similarly long times are considered for an ecological system, and each iteration represented a year, these transients could exist for periods longer than most temperate and polar terrestrial ecosystems have existed. In such a case, the transient is the only behaviour that is observed, and may be more persistent than the system itself.

Long lived transients, which are not chaotic, occur also in matrix models of population dynamics. In the matrix model considered in this paper, these 'transients' last from 200 to 1000 iterations (figure 1). In this case, the trajectory takes 10-20 iterations to reach a period one fixed point, and then no change is apparent for several hundred iterations on average, and then it moves quickly to a period two attractor. The important feature of this is that the qualitative behaviour, until the trajectory begins to move away from the period one fixed point, is indistinguishable.
from the behaviour shown when, with a small change in one parameter in the model, the trajectory approaches, and remains at, an asymptotically stable period one fixed point. That the transient in this model looks like an attractor that existed in a similar position with slightly different parameter values is what it has in common with the chaotic transients described by Grebogi, et al. (1986).

This creates a problem, if the transient exists for decades or centuries, for a field biologist who has to examine an ecosystem and determine whether or not it is stable, e.g. for the purpose of exploitation. It is unlikely that estimation of parameter values from data collected from an ecosystem will yield precision better than two or three significant figures, even with enormous (and expensive) datasets. Nor is it likely that the system will be observed for more than a few years. In such circumstances, how can an ecologist distinguish between a stable attractor and a 'transient'? When an ecosystem has apparently not changed greatly for a number of years (or decades or centuries), and then it changes rapidly to some other, apparently stable, state, it is usually assumed that some natural or anthropogenic catastrophe or pressure has caused the change. However, as the transients examined here show, there may have been no external change causing the observed behaviour. The observed behaviour may just be a long lived transient, and a search for a causal mechanism, such as over-exploitation or pollution of the system, or climatic change, may be futile.

**Semi-stable Attractors**

There are three reasons for defining the transients considered here to be semi-stable attractors. First, they derive from an attractor with a small change in a parameter. What has happened, in the case of the matrix population model for example, is that an equilibrium has been transformed into a saddle point as the parameter has passed through the catastrophe set. However, not all saddle points will display this behaviour. What is required is that the rate of change in the unstable manifold is substantially smaller than that in the stable manifolds: small enough that the absolute value of the determinant of the Jacobian is less than one. There is thus an intimate connection between semi-stable attractors and attractors in that the semi-stable attractor is a destabilized attractor.

![Figure 1. Age specific abundances through time. (Population size vs time.) After the bifurcation, after about 500 iterations, each age group oscillates between the two populations sizes indicated for it. Legend: □ - Age group 1, + - Age group 2, ♦ - Age group 3, ○ - Age group 4, * - Age group 5](image-url)