MIE SCATTERING NEAR THE CRITICAL ANGLE

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Abstract - Complex angular momentum theory is applied to the problem of high-frequency critical light scattering by a spherical cavity near the critical angle. The main contributions to the scattering arise from a critical domain close to critical incidence. The results are in good agreement with the exact Mie solution.

1. INTRODUCTION

The complex angular momentum (CAM) theory of Mie scattering and its application to the problems of the rainbow and the glory have been reviewed elsewhere. We present here the latest application of CAM theory: the treatment of critical scattering. This is a new diffraction effect found in the transition region around the critical scattering angle for refractive index $N$ relative to the external medium <1 (e.g., for an air bubble in water). The assumptions are $(k a)^{1/3} \gg 1$ and $(1-N)^{1/2}(k a)^{1/3} \gg 1$, where $k$ is the wavenumber in the external medium and $a$ is the radius of the cavity. The

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main contributions arise from a "critical domain" close to critical incidence, and they lead to a new kind of diffraction integral.

In Mie scattering for \( N < 1 \), the critically incident ray is reflected at a critical scattering angle

\[
\theta_t = \pi - 2\theta_c = \pi - 2\sin^{-1}N.
\]  

(1)

According to ray optics, total reflection takes place for angles of incidence beyond \( \theta_c \), i.e., for \( \theta > \theta_c \).

In the geometrical optics approximation \(^3\), the angular distribution of the scattered intensity goes through a cusp at \( \theta = \theta_t \). This singularity arises from the abrupt approach of the Fresnel reflectivities to unity at the critical angle \(^4\).

Exact Mie calculations \(^5\) show an oscillatory behavior of the intensity in the total reflection region near \( \theta_t \) (\( \theta < \theta_t \)). These diffraction fringes have also been observed experimentally \(^6\).

A "physical optics approximation" along the lines of classical diffraction theory has been proposed by Marston \(^6\). The contribution from surface reflection is treated by a procedure similar to Airy's theory of the rainbow: a Kirchhoff-type approximation is applied to the amplitude distribution along a virtual reflected wavefront. In view of their steep approach to total reflection, the reflectivities are approximated by step functions. This "reflectivity edge" gives rise to an angular distribution of scattered intensity similar to a Fresnel straight-edge pattern, which would account for the diffraction fringes.

The actual angular distribution \(^5\) differs from the Fresnel one: the oscillation amplitude increases as one goes farther away from \( \theta_t \). This reinforcement was explained \(^7\) through interference with directly transmitted rays due to below-critical incidence.

Superimposed on the "slow" oscillations just discussed, the Mie patterns \(^5\) show fine structure, represented by rapid oscillations of relatively smaller amplitude. This arises from interference with "far-side" contributions (in nuclear scattering terminology \(^8\), mainly from rays that have undergone one internal reflection. The fine structure is unrelated with critical scattering,