SCATTERING OF A GAUSSIAN BEAM BY A SPHERE USING A BROMWICH FORMULATION:

CASE OF AN ARBITRARY LOCATION

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I - INTRODUCTION

The present paper is devoted to the generalization of the Mie scattering theory for a sphere illuminated by a plane wave to the case when the scatter center is illuminated by a Gaussian beam. Such a fundamental theory may lead, in other steps, to important applications in optical sizing, by enabling the researchers to design rigorous approaches to the principles of a few optical sizing methods (the visibility or the phase Doppler techniques, for instance).

The problem of Mie theory generalization has been considered by several authors (2,3,4,5, among others), presenting theoretical approaches and numerical results of various extents. The work carried out in this field by our team traces back to 1980. Formal works have been published in a series of papers in the Journal of Optics 7,8,9 and numerical results appeared in Applied Optics 10,11, including the design of a so-called localized approximation (see ref 12 in the present symposium). However, these works were restricted to the case when the scatter center is located on the axis of the incident beam. The present work is devoted to a final generalization, the location of the scatter center being arbitrary. The formulation is not given for the cross-sections and pressure radiation for lack of room, although it has been established.

II - THE BROMWICH FORMULATION

To solve the Maxwell equations, taking into account boundary conditions, in a spherical coordinate system (r,θ,φ), fig 1, the Bromwich formulation 2,3,7,8 writes the solution as the sum of two special solutions: the Tranversal Magnetic (TM) wave for which \( H_\theta = 0 \) and the Transversal Electric (TE) wave for which \( E_\phi = 0 \). The TM and TE-fields are deduced from Bromwich Scalar Potentials (BSP), \( U_{TM} \) and \( U_{TE} \), respectively. Any BSP, \( U \), complies with the following equation:

\[
\frac{\partial^2 U}{\partial r^2} + k^2 U + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 U}{\partial \phi^2} = 0
\]  

(1)
in which \( k \) is the wave-number:

\[
  k = \omega \left( \frac{\mu \varepsilon}{c} \right)^{1/2} = M \frac{\omega}{c}
\]  

(2)

where \( \omega \) is the angular frequency of the electromagnetic sinusoidal wave varying in time like \( \exp(i\omega t) \), \( \mu \) and \( \varepsilon \) are respectively the permeability and the permittivity of the medium, \( M \) its complex refractive index and \( c \) the speed of light.

The following BSP:

\[
  U = \sum_{n=1}^{\infty} c_n \Psi_n(kr) P_n^m(\cos \theta) \begin{pmatrix} \sin \\ \cos \end{pmatrix}(m\psi)
\]

(3)

are solutions of (1). The \( \Psi_n(kr) \) are the spherical Bessel functions \(^7,8\) and \( P_n^m \) the associated Legendre polynomials \(^7,9\). The following BSP:

\[
  U = \sum_{n=1}^{\infty} c_n \varepsilon_n(kr) P_n^m(\cos \theta) \begin{pmatrix} \sin \\ \cos \end{pmatrix}(m\psi)
\]

(4)

are also solutions of (1). The \( \varepsilon_n(kr) \) are given by:

\[
  \varepsilon_n(kr) = \Psi_n(kr) + i (-1)^n \sqrt{\frac{\pi kr}{2}} J_{-n-1/2}(kr)
\]

(5)

in which \( \Psi_n(kr) \) are the Ricatti-Bessel functions, equal to \( kr \Psi_n^1(kr) \), and the \( J_{-n-1/2}(kr) \) are the Bessel functions of (negative) half-integer order. \( \Psi_n^1(kr) \) remains finite for \( r = 0 \) while \( \varepsilon_n(kr) \) tends to a spherical wave for \( r \to \infty \).

From the definition of the TM- and TE-waves:

\[
  H_{r,TM} - E_{r,TE} = 0
\]

(6)

When the BSP are determined, the other field components are obtained from the following relations:

\[
  E_{r,TM} = \frac{\partial^2 U_{TM}}{\partial r^2} + k^2 U_{TM}
\]

(7)

\[
  E_{\theta,TM} = \frac{1}{r} \frac{\partial^2 U_{TM}}{\partial r \partial \theta}
\]

(8)

\[
  E_{\phi,TM} = \frac{1}{r \sin \theta} \frac{\partial^2 U_{TM}}{\partial r \partial \phi}
\]

(9)

\[
  H_{\theta,TM} = \frac{i \omega \varepsilon}{r \sin \theta} \frac{\partial U_{TM}}{\partial \phi}
\]

(10)

\[
  H_{\phi,TM} = -\frac{i \omega \mu}{r} \frac{\partial U_{TM}}{\partial \theta}
\]

(11)

\[
  E_{\theta,TE} = -\frac{i \omega \mu}{r \sin \theta} \frac{\partial U_{TE}}{\partial \phi}
\]

(12)

\[
  E_{\phi,TE} = \frac{i \omega \mu}{r} \frac{\partial U_{TE}}{\partial \theta}
\]

(13)