8 / IIR Filter Design by Transformation

8.0 / Introduction

The first techniques for discrete-time filter design we shall study involve transformation of the designs for continuous-time filters. The classical filter approximations are of particular interest because they are equally applicable, in most cases, to continuous- or discrete-time design and because they are so widely known and satisfy various optimality criteria. Of necessity, discrete-time filters derived by transformation from continuous-time designs are IIR filters because the number of finite poles must equal or exceed the number of finite zeros in the continuous-time case in order to prevent infinite gain at infinite frequency.

The system function for a rational continuous-time filter is of the form

\[ H_c(s) = \frac{\sum_{m=0}^{M_c} c_m s^m}{\sum_{k=0}^{N} d_k s^k}, \quad (8.0.1) \]

where \( M_c \leq N \) to prevent poles at infinity, as mentioned above. As implied by the notation in (8.0.1), the order \( N \) of the denominator of the system function will, in general, be the same for corresponding continuous- and discrete-time filters, whereas the numerator polynomials can be of different orders depending upon the transformation used.

Various transformations can be chosen to preserve various properties of the continuous-time filter, but we will generally want the \( j\Omega \)-axis in the \( s \) plane to map into the unit circle in the \( z \) plane. We
will also insist that the left-half plane map into the interior of the
unit circle to preserve stability. We must emphasize at this point
that although many different transformations can be used for filter
design (as long as the above constraints are met), it is usually
meaningless to apply them to signals; only the standard $z$ transform
defined in chapter 3 is appropriate for signal analysis.

8.1 / Classical Filter Designs

Most often, transformation techniques are applied to the design of
discrete-time filters with classical continuous-time counterparts.
The classical filter designs of primary interest to us—namely, But­
terworth, Chebyshev, and elliptic—satisfy constraints on the mag­
nitude of the frequency response $H_e(j\Omega)$ of the form illustrated in
figure 8.1 for the lowpass case. That is, in the passband, the
frequency response is required to satisfy

$$1 \geq |H_e(j\Omega)| \geq 1 - \delta_1, \quad |\Omega| \leq \Omega_c,$$

and in the stopband

$$|H_e(j\Omega)| \leq \delta_2, \quad |\Omega| \geq \Omega_r,$$

Figure 8.1. Classical lowpass filter specifications for the magnitude re-
response.