Introduction

The Partial Credit Model (PCM) is a unidimensional model for the analysis of responses recorded in two or more ordered categories. In this sense, the model is designed for the same purpose as several other models in this book, including Samejima’s graded response model (Samejima, 1969). The PCM differs from the graded response model, however, in that it belongs to the Rasch family of models and so shares the distinguishing characteristics of that family: separable person and item parameters, sufficient statistics, and, hence, conjoint additivity. These features enable “specifically objective” comparisons of persons and items (Rasch, 1977) and allow each set of model parameters to be conditioned out of the estimation procedure for the other.

The PCM (Masters, 1982, 1987, 1988a, 1988b) is the simplest of all item response models for ordered categories. It contains only two sets of parameters: one for persons and one for items. All parameters in the model are locations on an underlying variable. This feature distinguishes the PCM from models that include item “discrimination” or “dispersion” parameters which qualify locations and so confound the interpretation of variables.

In this chapter, the PCM is introduced as a straightforward application of Rasch’s model for dichotomies (Rasch, 1960) to pairs of adjacent categories in a sequence. The simplicity of the model’s formulation makes it easy to implement in practice, and the model has been incorporated into a range of software packages.

The PCM can be applied in any situation in which performances on an item or an assessment criterion are recorded in two or more ordered categories and there is an intention to combine results across items/criteria to obtain measures on some underlying variable. Successful applications of the PCM to a wide variety of measurement problems have been reported in the literature. These include applications to the assessment of language functions in aphasia patients (Guthke et al., 1992; Willmes, 1992); ratings of infant performance (Wright and Masters, 1982); computerized patient simulation problems (Julian and Wright, 1988); ratings of writing samples (Pollitt and Hutchinson, 1987; Harris et al., 1988); measures of critical thinking (Masters and Evans, 1986); ratings of second language proficiency...
(Adams et al., 1987); computer adaptive testing (Koch and Dodd, 1989); answer-until-correct scoring (Wright and Masters, 1982); measures of conceptual understanding in science (Adams et al., 1991); embedded figure tests (Pennings, 1987); self-ratings of fear (Masters and Wright, 1982); applications of the SOLO taxonomy (Wilson and Iventosch, 1988); self-ratings of life satisfaction (Masters, 1985); the diagnosis of mathematics errors (Adams, 1988); measures of conceptual understanding in social education (Doig et al., 1994); the construction of item banks (Masters, 1984); and statewide testing programs (Titmanis et al., 1993).

Presentation of the Model

The PCM is an application of Rasch’s model for dichotomies. When an item provides only two scores 0 and 1, the probability of scoring 1 rather than 0 is expected to increase with the ability being measured. In Rasch’s model for dichotomies, this expectation is modeled as

$$\frac{P_{ij1}}{P_{ij0} + P_{ij1}} = \frac{\exp(\theta_j - \delta_i)}{1 + \exp(\theta_j - \delta_i)},$$

where $P_{ij1}$ is the probability of person $j$ scoring 1 on item $i$, $P_{ij0}$ is the probability of person $j$ scoring 0, $\theta_j$ is the ability of person $j$, and $\delta_i$ is the difficulty of item $i$ defined as the location on the measurement variable at which a score of 1 on item $i$ is as likely as a score of 0. The model is written here as a conditional probability to emphasize that it is a model for the probability of person $j$ scoring 1 rather than 0 (i.e., given one of only two possible outcomes and conditioning out all other possibilities of the person–item encounter such as “not answered” and “answered but not scorable”).

When an item provides more than two response categories (e.g., three ordinal categories scored 0, 1, and 2), a score of 1 is not expected to be increasingly likely with increasing ability because, beyond some point, a score of 1 should become less likely as a score of 2 becomes a more probable result. Nevertheless, it follows from the intended order $0 < 1 < 2, \ldots, < m_i$ of a set of categories that the conditional probability of scoring $x$ rather than $x - 1$ on an item should increase monotonically throughout the ability range. In the PCM, this expectation is modeled using Rasch’s model for dichotomies:

$$\frac{P_{ijx}}{P_{ijx-1} + P_{ijx}} = \frac{\exp(\theta_j - \delta_{ix})}{1 + \exp(\theta_j - \delta_{ix})}, \quad x = 1, 2, \ldots, m_i$$

where $P_{ijx}$ is the probability of person $j$ scoring $x$ on item $i$, $P_{ijx-1}$ is the probability of person $j$ scoring $x - 1$, $\theta_j$ is the ability of person $j$, and $\delta_{ix}$ is an item parameter governing the probability of scoring $x$ rather than $x - 1$ on item $i$. 