Chapter 10

Cointegration and the Aggregate Demand for Money Function

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We have argued that in a multivariate context with integrated variables it is important to test for cointegration (i.e., long-run equilibrium relationships). If the variables are integrated of the same order, but not cointegrated, ordinary least squares yields misleading results. In fact, Peter Phillips (1987) formally proves that a regression involving integrated variables is spurious in the absence of cointegration. In this case, the only valid relationship that can exist between the variables is in terms of their first differences.

However, if the variables are integrated and cointegrate, then there is a long-run equilibrium relationship between them. Moreover, the dynamics of the variables in the system can be described by an error correction model in which the short-run dynamics are influenced by the deviation from the long-run equilibrium. This is known as the ‘Granger representation theorem’ stating that for any set of integrated variables, cointegration and error correction are equivalent representations.

In this chapter we explore recent exciting developments in the field of applied econometrics, pertaining to the empirical analysis of models characterized by integrated and cointegrated variables. We begin with a brief review of recent theoretical developments and then discuss
empirical issues in modeling and estimating aggregate money demand functions.

10.1 Cointegration and Common Trends

Cointegration is a relatively new statistical concept, introduced into the economics literature by Engle and Granger (1987). It is designed to deal explicitly with the analysis of the relationship between integrated series. In particular, it allows individual time series to be integrated, but requires a linear combination of the series to be stationary. Therefore, the basic idea behind cointegration is to search for a linear combination of individually integrated time series that is itself stationary.

Consider the null hypothesis that there is no cointegration between two integrated series, \( y_t \) and \( x_t \), or equivalently, there are no shared stochastic trends (i.e., there are two distinct stochastic trends) between these series, in the terminology of Stock and Mark Watson (1988). The alternative hypothesis is that there is cointegration (or equivalently, \( y_t \) and \( x_t \) share a stochastic trend). Following Engle and Granger (1987), one can estimate the so-called \textit{cointegrating regression} (selecting arbitrarily a normalization)

\[
y_t = a + bx_t + \varepsilon_t. \tag{10.1}
\]

A test of the null hypothesis of no cointegration (against the alternative of cointegration) is based on testing for a unit root in the ordinary least squares (OLS) regression residuals \( \hat{\varepsilon}_t \).

10.2 Error Correction Models

If a cointegrating relationship is identified, for example \( \hat{\varepsilon}_t \) is integrated of order zero, then according to the Engle and Granger (1987) representation theorem there must exist an error correction representation relating current and lagged first differences of \( y_t \) and \( x_t \), and at least one lagged value of \( \hat{\varepsilon}_t \). In particular, in the present context the error correction model can be written as

\[
\Delta y_t = \alpha_1 + \alpha_{y,1} \hat{\varepsilon}_{t-1} + \sum_{j=1}^{r} \alpha_{11}(j) \Delta y_{t-j} + \sum_{j=1}^{s} \alpha_{12}(j) \Delta x_{t-j} + \varepsilon_{yt}, \tag{10.2}
\]

\[
\Delta x_t = \alpha_2 + \alpha_{x,1} \hat{\varepsilon}_{t-1} + \sum_{j=1}^{r} \alpha_{21}(j) \Delta y_{t-j} + \sum_{j=1}^{s} \alpha_{22}(j) \Delta x_{t-j} + \varepsilon_{xt}, \tag{10.3}
\]