Chapter 2

ARE THE UNIT-ROOT TESTS ADEQUATE FOR NONLINEAR MODELS?

1. Introduction

In the previous chapter, we surveyed some basic results achieved in the literature about nonstationary and nonlinear time series models. The motivation of this chapter is to consider these notions together. Indeed, as suggested in chapter 1, many economic time series display both nonlinear and nonstationary behavior and it seems a natural approach to consider them together. As a starting point, we ask the following question. Consider a nonstationary variable \( X_t \). What are the properties of the nonlinear model \( X_t = f(X_{t-1}, \ldots, X_{t-p}) + \varepsilon_t \) generating \( X_t \)? It is impossible to give a general answer without specifying the exact form of the function \( f \). In the sequel, we shall adopt an empirical approach and consider nonlinear processes currently used in the different fields of applied economics. However, this is not enough if we want to answer our question. We also need enquiring about the type of nonstationarity that is produced by nonlinear models. In general terms, the fact that a process has no stationary asymptotic distribution is due to several causes.

(a) The definition of nonstationarity implies that a given process has time-varying moments. This typical case occurs when, for instance, the mean and variance are functions of other stochastic variables. For nonlinear models, this phenomenon is known as "growth phenomenon" and a statistical theory was constructed during the beginning of the eighties by Kersting (1986) and Klebaner (1989). Growth models have received little attention in the economic field. This might be partly due to the fact that the literature on the linear approach of nonstationarity that has emerged during the same years seemed less complicated and posed itself
new problems. Growth theory is, however, a currently active research field in applied econometrics (see Granger et al. (1997) for economic applications).

(b) Nonstationarity is also associated to explosive dynamics. This is a common view that the polynomials corresponding to linear processes can have roots that are larger than 1 in modulus or that are located on the unit circle. The same requirements may well apply to nonlinear processes, particularly if we consider nonlinear extensions of ARMA models. With such a formulation, the terms "unit root" and "nonstationarity" are synonymous. Thereby, an interesting question may be the following: what are the consequences of the presence of unit roots in nonlinear models?

(c) Nonstationarity is also a feature of models with structural instability. There are processes with time-dependent polyspectra and for which it is hopeless to find an asymptotic stochastic distribution that is stationary. This can occur for time-varying parameter models, or regime-switching processes. Such processes received a great attention in different fields of natural sciences and gave rise to new developments such as the theory of wavelets, or the analysis of time-dependent polyspectra (see, among others, Page (1952), Priestley and Subba Rao (1968), Tjostheim (1976)). A promising area of application in economics might be finance and macroeconomics.

In this chapter, our definition of "nonstationarity" corresponds to the first two interpretations. We thus assume that a nonlinear process may be nonstationary, either because some of the roots of its characteristic polynomials are greater than or equal to 1 in modulus, or because its first two moments are time-varying. In regard to this, we address the following questions.

- Question 1. Suppose that the nonlinear process has some unit roots. Does this imply that it is truly nonstationary? This question implies some clarifications on the distinction between local nonstationarity and global nonstationarity. Such a distinction is needed, notably if the nonlinear model encompasses some linear models.

- Question 2. How can we empirically detect the presence of unit roots in nonlinear models? Do the usual linear approaches work? And does nonlinearity affect the distribution of the usual statistics?

- Question 3. It is an open problem whether nonlinearly transformed unit roots are still unit roots. This problem is important since, in many cases, economists use transformations on their data before applying unit root tests. For instance, financial economists