Transmission lines are used to transfer electrical signals (information) or electrical power from one point to another in an electrical system. Transmission lines take a wide variety of forms, from simple wire pairs and cables to more complicated integrated structures for high-frequency applications. Several common transmission lines are shown in Figure 7.1.

The chapter begins with a discussion of general transmission-line analysis, both from the standpoint of impedance methods and from the theory of Sturm–Liouville operators. Arbitrarily terminated transmission lines are treated using Green’s function techniques, and the spectral analysis of transmission-line operators is briefly discussed. Then, transmission-line resonators and unbounded transmission lines are treated using regular and singular Sturm–Liouville theory, respectively. The chapter concludes with a discussion of general multiconductor transmission lines.

Figure 7.1: Several types of two-conductor TEM transmission lines: (a) parallel wires, (b) coaxial cable, (c) parallel-plate waveguide.
7.1 General Analysis

To illustrate the analysis of transmission lines and transmission-line resonators, consider the two-conductor TEM transmission line depicted in Figure 7.2, where $V_s$ and $I_s$ represent distributed sources. The lumped-element model for a small segment of the line is shown in Figure 7.3. The circuit elements are

- $R$: series resistance per unit length for both conductors, ohms/m
- $L$: series inductance per unit length for both conductors, H/m
- $G$: shunt conductance per unit length, S/m
- $C$: shunt capacitance per unit length, F/m
- $i_s$: shunt current source per unit length, A/m
- $v_s$: series voltage source per unit length, V/m

The distributed sources may represent, for instance, distributed currents and voltages induced on the transmission line by an external source (see, e.g., [14, pp. 429–436]). If desired, a localized source can be modeled by $v_s = v_0 \delta(z - z')$, and similarly for $i_s$. With the exception of the distributed sources, the engineering analysis of this lumped-element circuit model is found in many books, [15, Ch. 2], [16, Ch. 3], and so here we will focus on considering the problem from the standpoint of Sturm–Liouville theory. We will assume $R, L, G, C \in \mathbb{R}$.

Applying Kirchhoff’s voltage law and current law to the circuit of Figure 7.3 yields, respectively,

$$v(z, t) + v_s(z, t)\Delta z - R \Delta z i(z, t) - L \Delta z \frac{\partial i(z, t)}{\partial t} - v(z + \Delta z, t) = 0$$

$^{1}$Generally the term “conductor” is used in transmission-line analysis, and, in fact, the lines are usually conductive. However, other transmission systems, including those only involving dielectrics (e.g., optical fibers), can be modeled using these techniques.