III SIGNAL DETECTION IN DISCRETE TIME

III.A Introduction

In Chapter II we discussed several basic optimality criteria and design methods for binary hypothesis-testing problems. In this chapter we apply these and related methods to derive optimum procedures for detecting signals embedded in noise. To avoid analytical complications, we consider exclusively the case of discrete-time detection, leaving the continuous-time case for Chapter VI. The discrete-time case is of considerable practical interest due to the trend of increased digitization of signal processing functions.

In Section III.B we discuss various models for signal detection problems and derive the resulting optimum detector structures corresponding to the criteria set forth in Chapter II. Section III.C deals with some methods of analyzing performance of these structures for situations in which the closed-form computation of relevant error probabilities is not tractable. There are several useful design methods for detection procedures other than those discussed in Chapter II, and in Sections III.D and III.E we introduce three such methods, namely sequential, robust, and nonparametric detection.

III.B Models and Detector Structures

The basic physical observation model that we wish to consider is that of an observed continuous-time waveform that consists of one of two possible signals corrupted by additive noise. Our objective is to decide which of the two possible signals is present, and we wish to do so by processing a finite number (say \( n \)) of samples taken from the observed waveform.
This problem can be modeled statistically by the following hypothesis pair for the observation space \((\Gamma, G) = (\mathbb{R}^n, B^n)\):

\[
H_0: Y_k = N_k + S_{0k}, \quad k = 1, 2, \ldots, n
\]

versus

\[
H_1: Y_k = N_k + S_{1k}, \quad k = 1, 2, \ldots, n,
\]

where \(Y = (Y_1, \ldots, Y_n)^T\) is an observation vector consisting of the samples from the observed waveform, \(N = (N_1, \ldots, N_n)^T\) is a vector of noise samples, and \(S_0 = (S_{01}, \ldots, S_{0n})^T\) and \(S_1 = (S_{11}, \ldots, S_{1n})^T\) are vectors of samples from the two possible signals.\(^\dagger\) Note that the interpretation of \(Y\) as a vector of time samples is not the only possibility for (III.B.1) since the same model also arises if, for example, we take simultaneous (in time) samples from \(n\) spatially separated signal sensors or from the outputs of a bank of \(n\) parallel filters. In any case, we will refer to this subscript as a time parameter, although the results of course apply equally well to other situations modeled by (III.B.1).

Optimum procedures for deciding between \(H_0\) and \(H_1\) can be derived using the results of Chapter II if we have models for the statistical behavior of the signals and noise. For practical purposes the signals \(S_0\) and \(S_1\) can usually be classified as one of three basic types. They can be completely known (i.e., deterministic), they can be known except for a set of unknown (possibly random) parameters, or they can be completely random and thus specified only by their probability distributions. Sometimes (e.g., in radar/sonar problems) one of the signals, usually \(S_0\), is identically zero, so that we are actually trying to detect a signal embedded in noise. For the purposes of this treatment we will assume that the noise is independent of the signals under each hypothesis and that its probability distribution does not depend on which hypothesis is true. This assumption is valid for most applications, although in some applications the noise can depend on the signal (an example of this is the radar/sonar problem, in which the noise is sometimes partially composed of spurious signal reflections from the ground or from objects other than

\(^\dagger\) Here, and elsewhere in this book, vectors are taken to be columnar and superscript \(T\) denotes transposition.