V ELEMENTS OF SIGNAL ESTIMATION

V.A Introduction

In Chapter IV we discussed methods for designing estimators for static parameters, that is, for parameters that are not changing with time. In many applications we are interested in the related problem of estimating dynamic or time-varying parameters. In the traditional terminology, a dynamic parameter is usually called a signal, so the latter problem is known as signal estimation or tracking.

Such problems arise in many applications. For example, one function of many radar systems is to track targets as they move through the radar’s scanning area. This means that the radar must estimate the position of the target (and perhaps its velocity) at successive times. Since the targets of interest are usually moving and the position measurements are noisy, this is a signal estimation problem. Another application is that of analog communications, in which analog information (e.g., audio or video) is transmitted by modulating the amplitude, frequency, or phase of a sinusoidal carrier. The receiver’s function in this situation is to determine the transmitted information with as high a fidelity as possible on the basis of a noisy observation of the received waveform. Again, since the transmitted information is time varying, this problem is one of signal estimation.

The dynamic nature of the parameter in signal estimation problems adds a new dimension to the statistical modeling of these problems. In particular, the dynamic properties of the signal (i.e., how fast and in what manner it can change) must be modeled at least statistically in order to obtain meaningful signal estimation procedures. Also, performance expectations for estimators of dynamic parameters should be different from those for static parameters. In particular, unlike the static case, we cannot expect an estimator of a signal to be
perfect as the number of observations becomes infinite because of the time variation in the signal.

In this chapter we discuss the basic ideas behind some of the signal estimation techniques used most often in practice. In Section V.B we discuss Kalman-Bucy filtering, which provides a very useful algorithm for estimating signals that are generated by finite-dimensional linear dynamical models. In Section V.C the general problem of estimating signals as linear transformations of the observations is developed, and in Section V.D a particular case of linear estimation, Wiener-Kolmogorov filtering, which is a method of estimating signals whose statistics are stationary in time, is considered.

V.B Kalman-Bucy Filtering

Many time-varying physical phenomena of interest can be modeled as obeying equations of the type

$$X_{n+1} = f_n(X_n, U_n), \quad n = 0, 1, ..., \quad (V.B.1)$$

where $X_0, X_1, ...$ is a sequence of vectors in $\mathbb{R}^m$ representing the phenomenon under study; $U_0, U_1, ...$ is a sequence of vectors in $\mathbb{R}^s$ "acting" on $\{X_n\}_{n=1}^\infty$; and where $f_0, f_1, ...$ is a sequence of functions (or, in other words, a time-varying function), each mapping $\mathbb{R}^m \times \mathbb{R}^s$ to $\mathbb{R}^m$. Equation (V.B.1) is an example of a dynamical system, with $X_n$ representing the state of the system at time $n$ and with $U_n$ representing the input to the system at time $n$. A dynamical system is a system having the property that for any fixed times $l$ and $k$, $X_l$ is determined completely from the state at time $k$ (i.e., $X_k$) and the inputs from times $k$ up through $l-1$ (i.e., $\{U_n\}_{n=k}^{l-1}$). Note that complete determination of $\{X_n\}_{n=1}^\infty$ from (V.B.1) requires not only the specification of the input sequence but also the specification of the initial condition $X_0$. If the input sequence or the initial condition is random, the states $X_0, X_1, ...$ form a sequence of random vectors and (V.B.1) is referred to as stochastic system.

Equation (V.B.1) describes the evolution of the states of a system, so it is usually known as the state equation of the system. The system may also have associated with it on output sequence $Z_0, Z_1, ...$ of vectors in $\mathbb{R}^k$, possibly different