VI SIGNAL DETECTION IN CONTINUOUS TIME

VI.A Introduction

In the preceding chapters we have presented the basic principles of signal detection and estimation, assuming throughout that our observation set is either a set of vectors or is a discrete set. Throughout this analysis a key role was played by a family of densities \( \{ p_{\theta}; \theta \in \Lambda \} \) on the observation space, either through the likelihood ratio in hypothesis testing, through the computation of an \textit{a posteriori} parameter distribution in Bayesian estimation, or through the study of MVUEs and MLEs in nonBayesian parameter estimation. This necessity of specifying a family of densities on the observation space is the primary reason for restricting our observation sets in the way that we have done. In particular, as we have seen, all the problems considered thus far have been treated using the ordinary probability calculus of probability density functions and probability mass functions.\(^\dagger\)

Although the observation sets treated thus far are of considerable interest in practice, there are many applications in which our observations are best modeled as a continuous-time random process. That is, our overall observation \( Y \) is a collection of random variables \( \{ Y_t; t \in [0, T] \} \) indexed by a continuous parameter \( t \), where for convenience we have chosen our observation interval to be \([0, T]\) for some \( T > 0 \). In this chapter and the following one, we consider signal detection and estimation problems with this type of observations. Signal detection is treated in this chapter, with signal estimation

\(^\dagger\) An exception is the linear estimation problem treated in Section V.D. Since we needed only a second-order statistical description for this problem, we were in fact able to extend our observation set to include sets of infinite sequences.
being treated in Chapter VII.

In continuous-time problems, the observation set $\Gamma$ becomes a set each of whose elements is a continuous-time waveform. Such a set is called a function space. In order to model signal detection and estimation problems in this setting, we need to construct families of densities on such sets. Since a density is a function that can be integrated (or summed) to give probabilities, the notion of a density in continuous time requires a method of integration on function spaces. This type of integration requires analytical techniques beyond those of ordinary calculus, and thus before treating signal detection and estimation problems in continuous time we must first develop some analytical tools for dealing with such problems.

In Section VI.B we discuss very briefly the theory of integration in abstract spaces. The purpose of this treatment is not to provide the reader with the details of this theory but rather to indicate how the notion of a density can be extended to function spaces, and in turn how the necessary modeling for continuous time problems can be accomplished. We also consider in Section VI.B a representation for continuous time random processes (the Karhunen-Loève expansion), which allows the reduction of such processes to equivalent discrete-time processes. This representation is the key to the solution of the signal detection problems presented in the remainder of the chapter. In Sections VI.C and VI.D we turn to specific problems of signal detection with continuous time observations. The theory of such problems is no different from that of the previous chapters once the appropriate families of densities have been specified. Thus these sections are concerned primarily with specific methods for finding appropriate density classes for models of interest in applications, although the systems and performance aspects of the resulting procedures are also discussed. Section VI.C is concerned with the problem of detecting deterministic (i.e., coherent) signs in Gaussian noise, and Section VI.D with the detection of stochastic signals in Gaussian noise.