WAVES IN WARM PLASMAS

1. INTRODUCTION

In the previous chapter we analyzed the characteristics of wave propagation in a cold plasma. Now we will extend the theory already developed to include the pressure gradient term in the momentum equation. We shall consider wave propagation in a warm electron gas (neglecting ion motion) and, also, in a fully ionized warm plasma (considering electrons and only one type of ions), in the absence as well as in the presence of an externally applied magnetic field.

2. WAVES IN A FULLY IONIZED ISOTROPIC WARM PLASMA

2.1 Derivation of the Equations for the Electron and Ion Velocities

Consider now a fully ionized warm plasma, composed of electrons and only one ion species, with no externally applied magnetic field. The equations of conservation of mass and of momentum, for the electrons and for the ions, can be written as

\[
\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0 \tag{2.1}
\]

\[
m_\alpha \frac{D \mathbf{u}_\alpha}{Dt} = q_\alpha (\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) - \frac{1}{n_\alpha} \nabla p_\alpha - m_\alpha \nu_{\alpha \beta}(\mathbf{u}_\alpha - \mathbf{u}_\beta) \tag{2.2}
\]
where for the electrons $\alpha = e$ and $\beta = i$ while for the ions $\alpha = i$ and $\beta = e$. These equations are complemented by the following adiabatic energy equation for each species,

$$p_\alpha n_\alpha^{-\gamma} = constant$$

(2.3)

where $\gamma = 1 + 2/N$ is the adiabatic constant and $N$ denotes the number of degrees of freedom. Applying the $\nabla$ operator to (2.3) and using the ideal gas law $p_\alpha = n_\alpha kT_\alpha$, we can rewrite (2.3) in the form

$$\nabla p_\alpha = \gamma k_BT_\alpha \nabla n_\alpha$$

(2.4)

We restrict our attention to small amplitude waves in order to linearize the equations and assume that

$$n_\alpha(r, t) = n_0 + n'_\alpha \exp \left( ik \cdot r - i\omega t \right) ; \quad |n'_\alpha| \ll n_0$$

(2.5)

$$u_\alpha(r, t) = u_\alpha \exp \left( ik \cdot r - i\omega t \right) ; \quad u_\alpha \ll |\omega/k|$$

(2.6)

$$E(r, t) = E \exp \left( ik \cdot r - i\omega t \right)$$

(2.7)

$$B(r, t) = B \exp \left( ik \cdot r - i\omega t \right)$$

(2.8)

Using these expressions in (2.1) and neglecting second-order terms, we find

$$\frac{n'_\alpha}{n_0} = \frac{1}{\omega} (k \cdot u_\alpha)$$

(2.9)

Similarly, we obtain for (2.2) after the substitution of $\nabla p_\alpha$ from (2.4) and linearizing,

$$-i\omega u_\alpha = \frac{q_\alpha}{m_\alpha} E - V_{\alpha\alpha}^2 ik \frac{n'_\alpha}{n_0} - \nu_{\alpha\beta}(u_\alpha - u_\beta)$$

(2.10)

where $V_{\alpha\alpha} = (\gamma k_BT_\alpha/m_\alpha)^{1/2}$ is the adiabatic sound speed for the type $\alpha$ particles. Substituting (2.9) into (2.10) and multiplying by $i\omega$, we obtain the following equation involving the variables $u_\alpha$, $u_\beta$, and $E$,

$$\omega^2 u_\alpha = i\omega \frac{q_\alpha}{m_\alpha} E + V_{\alpha\alpha}^2 k(k \cdot u_\alpha) - i\omega \nu_{\alpha\beta}(u_\alpha - u_\beta)$$

(2.11)

A relationship between the electric field and the electron and ion velocities can be obtained from Maxwell curl equations with harmonic variations of $E$ and $B$, according to (2.7) and (2.8),

$$k \times E = \omega B$$

(2.12)