As mentioned in Chapter 1, an important element of many decision problems is the prior information concerning \( \theta \). It was stated that a convenient way to quantify such information is in terms of a probability distribution on \( \Theta \). In this chapter, methods and problems involved in the construction of such probability distributions will be discussed.

3.1. Subjective Probability

The first point that must be discussed is the meaning of probabilities concerning events (subsets) in \( \Theta \). The classical concept of probability involves a long sequence of repetitions of a given situation. For example, saying that a fair coin has probability \( \frac{1}{2} \) of coming up heads, when flipped, means that, in a long series of independent flips of the coin, heads will occur about \( \frac{1}{2} \) of the time. Unfortunately, this frequency concept won’t suffice when dealing with probabilities about \( \theta \). For example, consider the problem of trying to determine \( \theta \), the proportion of smokers in the United States. What meaning does the statement \( P(0.3 < \theta < 0.35) = 0.5 \) have? Here \( \theta \) is simply some number we happen not to know. Clearly it is either in the interval \((0.3, 0.35)\) or it is not. There is nothing random about it. As a second example, let \( \theta \) denote the unemployment rate for next year. It is somewhat easier here to think of \( \theta \) as random, since the future is uncertain, but how can \( P(3\% < \theta < 4\%) \) be interpreted in terms of a sequence of identical situations? The unemployment situation next year will be a unique, one-time event.

The theory of subjective probability has been created to enable one to talk about probabilities when the frequency viewpoint does not apply. (Some
even argue that the frequency concept never applies, it being impossible to have an infinite sequence of i.i.d. repetitions of any situation, except in a certain imaginary (subjective) sense.) The main idea of subjective probability is to let the probability of an event reflect the personal belief in the "chance" of the occurrence of the event. For example, you may have a personal feeling as to the chance that \( \theta \) (in the unemployment example) will be between 3% and 4%, even though no frequency probability can be assigned to the event. There is, of course, nothing terribly surprising about this. It is common to think in terms of personal probabilities all the time; when betting on the outcome of a football game, when evaluating the chance of rain tomorrow, and in many other situations.

The calculation of a frequency probability is theoretically straightforward. One simply determines the relative frequency of the event of interest. A subjective probability, however, is typically determined by introspection. It is worthwhile to briefly discuss techniques for doing this.

The simplest way of determining subjective probabilities is to compare events, determining relative likelihoods. Say, for example, that it is desired to find \( P(E) \). Simply compare \( E \) with, say, \( E^c \) (the complement of \( E \)). If \( E \) is felt to be twice as likely to occur as \( E^c \), then clearly \( P(E) = \frac{2}{3} \) and \( P(E^c) = \frac{1}{3} \). This is rather loosely stated, but corresponds, we feel, to the intuitive manner in which people do think about probabilities. As with utility theory, a formal set of axioms can be constructed under which subjective probabilities can be considered to exist and will behave in the fashion of usual probabilities. (See DeGroot (1970) for one such system, and references to others.) Such systems show that the complexity of probability theory is needed to quantify uncertainty (i.e., nothing simpler will do), and at the same time indicate that it is not necessary to go beyond the language of probability.

An alternate characterization of subjective probability can be achieved through consideration of "betting" (and the related use of scoring rules, cf. deFinetti (1972) and Lindley (1982a)). In the betting scenario, one determines \( P(E) \) by imagining being involved in a gamble wherein \( z \) will be lost if \( E \) occurs and \( (1 - z) \) will be gained if \( E^c \) occurs, where \( 0 \leq z \leq 1 \). The idea is then to choose \( z \) so that the gamble is "fair" (i.e., has overall utility zero), resulting in the equation

\[
0 = \text{Expected Utility of the Gamble} = U(-z)P(E) + U(1-z)(1-P(E)).
\]

Solving for \( P(E) \) yields

\[
P(E) = \frac{U(1-z)}{[U(1-z) - U(-z)]}.
\]

If \( z \) is "small," \( U \) is probably approximately linear, so that \( P(E) \approx 1 - z \). One might object that this betting mechanism is circular, since utility functions were constructed by considering probabilistic bets, and now we are trying to determine \( P(E) \) from knowledge of the utility function. The dilemma can be resolved by noting that, in the construction of the utility