Chapter 7

Recursive Structures: Growing of Fractals and Plants

The development of an organism may [...] be considered as the execution of a ‘developmental program’ present in the fertilized egg. The cellularity of higher organisms and their common DNA components force us to consider developing organisms as dynamic collections of appropriately programmed finite automata. A central task of developmental biology is to discover the underlying algorithm from the course of development.

Aristid Lindenmayer and Grzegorz Rozenberg

The historical constructions of fractals by Cantor, Sierpinski, von Koch, Peano, etc., have been labeled as ‘mathematical monsters’. Their purpose had been mainly to provide certain counterexamples, for example, showing that there are curves that go through all points in a square. Today a different point of view has emerged due to the ground-breaking achievements of Mandelbrot. Those strange creations from the turn of the century are anything but exceptional counterexamples; their features are in fact typical of nature. Consequently, fractals are becoming essential components in the modeling and simulation of nature. Certainly, there is a great difference between the basic fractals shown in this book and their counterparts in nature: mountains, rivers, trees, etc. Surely, the artificial fractal mountains produced today in computer graphics already look stunningly real. But on the other hand they still lack something we would certainly feel while actually camping in the real mountains. Maybe it is the (intentional) disregarding of all developmental processes in the fractal models which is one of the factors responsible for this shortcoming.

Fractals in nature are always a result of some growth process. In contrast, we have regarded fractals as static. Indeed, it was a goal to characterize fractals as solutions to equations. And nothing can be more eternal than the solution of an equation. You may argue on the other hand that these fractals have usually been obtained from dynamical processes such as the one in the Multiple Reduction Copy Machine introduced in chapter 1. However, our attention has always been focused on the end result, the attractor of the machine. The process leading to this end product was important merely when it revealed properties of the attractor. In this chapter we pay more attention to the intermediate stages in the production of a fractal. We begin the discussion with another dialect of the language of fractals which has been created specifically for the description of natural growth processes. This dialect is called L-systems.

In figure 7.1 we illustrate the contrast between the way an MRCM may generate a twig from a bush or tree and how one would actually expect a real twig to grow with time.