Chapter 1

THE THRESHOLD ACCEPTING HEURISTIC FOR INDEX TRACKING*

Manfred Gilli
Department of Econometrics,
University of Geneva,
1211 Geneva 4, Switzerland
Manfred.Gilli@metri.unige.ch

Evis Këllezi
Department of Econometrics and FAME,
University of Geneva,
1211 Geneva 4, Switzerland
Evis.Kellezi@metri.unige.ch

Abstract We investigate the performance of the threshold accepting heuristic for the index tracking problem. The index tracking problem consists in minimizing the tracking error between a portfolio and a benchmark. The objective is to replicate the performance of a given index upon the condition that the number of stocks allowed in the portfolio is smaller than the number of stocks in the benchmark index. Transaction costs are incurred each time that the portfolio is rebalanced.

We find the composition of a portfolio that tracks the performance of the benchmark during a given period in the past and compare it with the performance of the portfolio in a subsequent period. We report computational results in the cases where the benchmarks are market indices tracked by a small number of assets. We find that the threshold accepting heuristic is an efficient optimization technique for this problem.

Keywords: Threshold Accepting, Heuristic Optimization, Index Tracking, Passive Fund Management

*We thank Peter Winker, Nick Webber and Agim Xhaja for insightful comments. Financial support from the Swiss National Science Foundation (project 12–5248.97) is gratefully acknowledged.
1. Introduction

Approaches to portfolio management can be divided into two broad categories – active and passive. Active strategies rely on the belief that skillful investors can out-perform the market by exercising activities such as market timing and stock picking. In recent years passive investment strategies have become very popular, especially among mutual fund managers and pension funds. These strategies are adopted by investors who believe that financial markets are efficient and it is therefore impossible to consistently beat the aggregate market return.

A passive strategy that attempts to reproduce as closely as possible the performance of a theoretical index representing the market, is called an index tracking strategy. Index tracking can be done by investing in all constituents of the index proportional to their share in the index, or by selecting a smaller subset of the assets in the index such that the resulting portfolio optimally (by some criteria) tracks the performance of the chosen index. This second approach is called partial replication and is generally preferred in practice, given that full replication involves high transaction costs and difficulties in rebalancing the portfolio when the weights in the tracked index change.

Despite the increasing popularity of passive investment strategies, the attention given in the academic literature to implementation and to algorithmic problems arising in the process of index tracking is relatively small compared to the numerous articles dedicated to the classical problem of portfolio risk and return optimization.

The problem is often formulated in such a way that classical techniques like quadratic or linear programming can be directly applied. The formulation of the problem involves two main choices. First of all, one must choose an objective function that is an appropriate function of the tracking error. This is usually specified as a measure of the closeness of the solution returns to the returns of the index. Another important choice is the set of constraints imposed on the solution. A realistic formulation of the problem should include restrictions on the positions on each asset, the number of assets in the portfolio, the size of transactions costs and minimum transaction lots, as well as liquidity and exposure constraints. The only way of handling them in the context of a realistic problem size is to use heuristic algorithms that provide good approximations of the optimal solution (see e.g. [3], [25], [7], [15], [24], [5], [19]).

Different approaches have been proposed in the literature. In the majority of the work related to index tracking ([30, 14, 28, 11, 8, 9, 17, 18, 23, 27, 24]), the tracking error is defined as the variance of the difference between tracking portfolio return and index return.

Another approach is used by Worzel et al. [32], Consiglio and Zenios [10] and Rudolf et al. [29], more recently by Rockafellar and Uryasev [26] and Konno