AN APPROACH TO CONSTRUCTING GENERALIZED PENALTY FUNCTIONS

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Abstract: We propose a general scheme of reduction of a problem of constrained minimization to a problem of unconstrained minimization for which an increasing function is used in order to construct a penalty function or a modified Lagrange function. The conditions of the equivalence of the initial and the auxiliary problems are given.

Key words: auxiliary function, generalized penalty function, modified Lagrange function.

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1 INTRODUCTION

One of the most important optimization problems is a problem of minimizing a non-linear function over a feasible set defined by a finite number of nonlinear inequality and/or equality constraints. There exist a large number of different algorithms for solving this problem. However, they can be divided into two major groups of methods. The first group consists of primal methods in

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which at each iteration a feasible descent direction is found and a step is being done along this direction. These methods include conditional gradient, reduced gradient methods, Zoutendijk's method, gradient projection method etc. (see Minoux (1989) and references therein for a detailed survey). However, they have many drawbacks involving their speed of convergence and the difficulty of the parameter adjustment.

Therefore in many cases it is preferable to use the dual methods which compose the second group. These methods include penalty function methods (see Fiacco and McCormick (1968), method of centers (see Huard (1967); Zabotin (1975)) and modified Lagrange methods (see Minoux (1989); Grossman and Kaplan (1979) and references therein). They give the information about the dual problem and thus make it possible to find lower estimates of the optimal value of the objective function on the feasible set which is very important for minimizing non-convex functions. Often these algorithms are called sequential unconstrained minimization techniques because they are based on the reduction of an initial constrained program to a sequence of problems of unconstrained minimization. Sometimes it is necessary to solve the auxiliary unconstrained problems many times. However, there exist exact penalty function methods in which it is sufficient to solve the auxiliary problem only once in order to obtain an optimal solution of the initial problem.

In this paper an approach to constructing the auxiliary functions for the dual methods is proposed which is based on increasing functions. We prove the equivalence of the initial and the auxiliary problems and show that many known concrete examples of the penalty or modified Lagrange functions fall within our general scheme which leads to constructing a family of new sequential unconstrained minimization methods. In the first section we consider the functions of penalty type for inequality constraints and we show that when the values of penalty coefficients are sufficiently large, we can obtain a solution of a constrained program within any required precision. In the second section we develop the same approach for the problems with equality constraints.

Usually it is preferable to use modified Lagrange functions rather than penalty functions, as the auxiliary problem can be ill-conditioned for large values of penalty parameters (for the discussion on modified Lagrange functions see Bertsekas (1982); Minoux (1989) and references therein). So in the third section we propose a scheme of generating modified Lagrange functions which allows to find an approximate optimal solution of the initial problem even if there are no saddle points of the Lagrange function. One of the promising possibilities is to combine the classical Lagrange approach with the method of centers and to apply known methods of nonsmooth optimization to solving the auxiliary problem.

2 GENERALIZED PENALTY FUNCTIONS FOR INEQUALITY CONSTRAINTS

Let us consider the following problem of constrained minimization: