INTRODUCTION

The formal theories considered here are all subsystems of second order arithmetic with the full comprehension principle, briefly (CA). They are theories for classical analysis: Hilbert used a theory equivalent to (CA) as a formal framework for mathematical analysis in lectures during the early twenties; an extensive portion of analysis had already earlier been developed by Weyl in a weak subsystem of (CA). Weyl's work aimed at rebuilding parts of analysis on a "sound" basis, and that meant, above all, avoiding impredicative principles. Hilbert, in contrast, set himself the task of securing the instrumental usefulness of all of classical mathematics. He hoped to achieve that aim by reducing analysis and even set theory to a fixed, absolutely fundamental part of arithmetic, so-called finitist mathematics. The specific proposal of how to achieve such a reduction is the mathematical centerpiece of Hilbert's program; it was refuted by Gödel's Incompleteness Theorems.

A generalized reductive program has been pursued for analysis, and significant progress has been made. Indeed, progress has been
made in two complementary directions. On the one hand, the work of Weyl and of other predicatively or constructively inclined mathe-
maticians has been extended to show that all of classical analysis
can be carried out in theories that are reducible to elementary
arithmetic. On the other hand, strong impredicative subsystems of
(CA) have been reduced to constructively acceptable theories of in-
ductive definitions. These results lead naturally to a distinction
between two types of reductions. The two types are distinguished
from each other not by the techniques for obtaining them, but rather
by programmatic aims. FOUNDATIONAL REDUCTIONS are to provide a con-
structive basis for strong and (from certain foundational perspect-
ives) problematic parts of (CA); COMPUTATIONAL REDUCTIONS are to
yield algorithmic information from proofs in weak and (even from a
finitist standpoint) unproblematic parts of (CA).

It is via foundational reductions that the generalized program
is pushed forward in an attempt to answer the question "WHAT MORE
THAN FINITIST MATHEMATICS DO WE HAVE TO KNOW TO RECOGNIZE THE (PART-
IAL) SOUNDNESS OF A STRONG THEORY?". Computational reductions answer
in a specific way Kreisel's question "WHAT MORE THAN ITS TRUTH DO
WE KNOW, IF WE HAVE PROVED A THEOREM BY RESTRICTED MEANS (HERE: IN
A WEAK SUBSYSTEM)?". Answers to the latter question are of mainly
mathematical interest, whereas answers to the former question are of
more philosophical significance: in any event, they provide detailed
material for reflections on the epistemology of mathematics.

A concise formulation of the generalized reductive program is
given below, and the aims of proof-theoretic investigations are dis-
cussed in greater detail. Some of the results which have been obtained
more recently are described, and there is even a new theorem or two.
However, the paper is expository and reflective. I hope it will be in-
formative and accessible to non-proof-theorists, as I believe that the
broad themes are, or should be, of general interest to philosophers