VELOCITY OF PROPAGATION OF GRAVITATIONAL RADIATION, MASS OF THE GRAVITON, RANGE OF THE GRAVITATIONAL FORCE, AND THE COSMOLOGICAL CONSTANT

J. Weber
University of Maryland
College Park, Maryland 20742
and
University of California
Irvine, California 92717

ABSTRACT

Einstein's equations with cosmological constant are considered. With an appropriate set of coordinates, the vacuum equations have the same form as the Klein Gordon Equation.

The range of the gravitational force is the Compton Wavelength of the graviton with rest mass m. The Cosmological Constant is one half the reciprocal of the squared Compton Wavelength.

Limits on the graviton mass are obtained by considering the observational data on the advance of the perihelion of Mercury, the observed gravitational radiation from Supernova 1987A, and the known gravitational binding of clusters of galaxies. The observed pulses from Supernova 1987A are in good agreement with the cross section theory published in 1984 and 1986, and reviewed here.

As first discussed by F. Zwicky, the graviton mass, as deduced from known gravitational binding of clusters of galaxies, is less than $1.2 \times 10^{-63}$ grams. The Cosmological Constant is less than $6.4 \times 10^{-52} \text{ cm}^{-2}$.

INTRODUCTION

Einstein's equations are

$$\gamma_{\nu} = \frac{1}{2} g_{\nu} R - \lambda g_{\nu} = \frac{8\pi G}{c^4} T_{\nu}$$

(1)
In (1) \( R_{\gamma \nu} \) is the Ricci tensor, \( R \) is the curvature scaler, \( g_{\gamma \nu} \) is the metric tensor, \( \lambda \) is the Cosmological Constant, \( T_{\gamma \nu} \) is the stress energy tensor, \( G \) is Newton's constant of gravitation, and \( c \) is the speed of light.

**WEAK FIELD SOLUTIONS**

For weak fields, let

\[
g_{\gamma \nu} = \delta_{\gamma \nu} + h_{\gamma \nu}
\]

(2)

\( \delta_{\gamma \nu} \) is the Lorentz metric and \( h_{\gamma \nu} \) is a first order quantity. To first order, the Ricci tensor may be written as

\[
R_{\gamma \nu} = - \frac{1}{2} \delta^{\sigma \alpha} h_{\gamma \nu, \sigma \alpha} - \frac{1}{2} \left[ \left( \frac{1}{2} \delta^{\beta}_\gamma h - h^{\beta}_\gamma \right) \rho_\nu + \left( \frac{1}{2} \delta^{\beta}_\nu h - h^{\beta}_\nu \right) \rho_\gamma \right]
\]

(3)

Coordinates are chosen such that

\[
\left( h^{\beta}_\gamma - \frac{1}{2} \delta^{\beta}_\gamma h \right) \rho_\beta = 0
\]

(4)

In (4) \( h \) is the trace \( h_{\alpha \alpha} \). The Ricci tensor is then given by

\[
R_{\gamma \nu} = - \frac{1}{2} \delta^{\sigma \alpha} h_{\gamma \nu, \sigma \alpha}
\]

(5)

Einstein's equations (1) are in this approximation

\[
- \frac{1}{2} \delta^{\sigma \alpha} h_{\gamma \nu, \sigma \alpha} - \frac{1}{2} g_{\gamma \nu} \left( - \frac{1}{2} \delta^{\sigma \lambda} h_{\nu, \sigma \lambda} \right) - \lambda g_{\gamma \nu} = \frac{8\pi G}{c^4} T_{\gamma \nu}
\]

(6)

In this order (6) may be written as

\[
- \frac{1}{2} \delta^{\sigma \alpha} \left( h_{\gamma \nu} - \frac{1}{2} \delta_{\gamma \nu} h \right), \sigma \alpha - \lambda g_{\gamma \nu} = \frac{8\pi G}{c^4} T_{\gamma \nu}
\]

(7)

New field quantities \( \phi_{\gamma \nu} \) and \( \phi_{\gamma} = \phi \) are defined by

\[
\phi_{\gamma \nu} = h_{\gamma \nu} - \frac{1}{2} \delta_{\gamma \nu} h
\]

(8)

\[
\phi_{\gamma} = \phi = h - 2 h = -h
\]

(9)

\[
\phi_{\gamma \nu} = g_{\gamma \nu} - \delta_{\gamma \nu} - \frac{1}{2} \delta_{\gamma \nu} h = g_{\gamma \nu} - \delta_{\gamma \nu} + \frac{1}{2} \delta_{\gamma \nu} \phi
\]

(10)

In terms of \( \phi_{\gamma \nu} \), Einstein's equations are

\[
- \frac{1}{2} \Box \phi_{\gamma \nu} - \lambda g_{\gamma \nu} = \frac{8\pi G}{c^4} T_{\gamma \nu}
\]

(11)

Raising one index and summing gives

\[
- \frac{1}{2} \Box \phi - 4\lambda = \frac{8\pi G}{c^4} T
\]

(12)