11 Approximate system reliability evaluation

11.1 Introduction

As discussed in Chapters 8 to 10, the Markov technique and the frequency and duration approach form sound and precise modelling and evaluation methods for reliability applications. They become less amenable, however, for hand calculations and even for digital computer solutions as the system becomes larger and more complex. In such cases, alternative methods are available which are based on the Markov approach and which use a set of appropriate but approximate equations. The essence of these approximate techniques is to derive a set of equations suitable for a series system in which all components must operate for system success and for a parallel system in which only one component need work for system success. These equations can then be used in conjunction with the network modelling techniques described in Chapters 4 and 5 to give rapid and sufficiently accurate results for a wide range of practical systems. In addition they are ideally suited for both hand calculations and digital computer implementation.

In this chapter the basic sets of equations are initially derived and used. These are then extended to more complex systems and applications.

11.2 Series systems

Consider the case of two components connected in series. The state space diagram for this system is shown in Figure 10.2 assuming that all states can exist. The probability of the system being in the up state, i.e., both components operating is given by Equation 10.7a; that is

\[ P_{up} = \frac{\mu_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \]  

(11.1)

where \( \lambda_1, \lambda_2 \) and \( \mu_1, \mu_2 \) are the failure rates and repair rates of the two components, respectively.

It is necessary to find the failure and repair rates, \( \lambda_s \) and \( \mu_s \), of a single component that is equivalent to the two components in series. This
Approximate system reliability evaluation

Fig. 11.1 Representation of a two component series system

is shown in Figure 11.1. The probability of the single component being in the up state is

\[ P_{\text{up}} = \frac{\mu_s}{\lambda_s + \mu_s} \]  

(11.2)

For the single component to be equivalent to the two series components, Equations 11.1 and 11.2 must be identical, i.e.,

\[ \frac{\mu_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} = \frac{\mu_s}{\lambda_s + \mu_s} \]  

(11.3)

Also, since the transition rate from the system up state, for the single equivalent component, is \( \lambda_s \), and for the two component series system, is \( \lambda_1 + \lambda_2 \), then

\[ \lambda_s = \lambda_1 + \lambda_2 \]  

(11.4)

Substituting Equation 11.4 into 11.3 and replacing the repair rates, \( \mu_i \), by the reciprocal of the average repair times, \( r_i \), gives

\[ r_s = \frac{1}{\mu_s} = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_1 \lambda_2 r_1 r_2}{\lambda_s} \]  

(11.5)

In many systems the product \( \lambda_i r_i \) is very small and therefore \( \lambda_1 \lambda_2 r_1 r_2 \ll \lambda_1 r_1 \) and \( \lambda_2 r_2 \). In such cases Equation 11.5 reduces to

\[ r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2}{\lambda_s} \]  

(11.6)

It should be noted that, although Equation 11.6 is an approximation for a two component series system in which all four states of Figure 10.2 exist, it is an exact expression for the situation in which state 4 of Figure 10.2 does not exist, i.e., when one component has failed, the second component cannot fail. This occurs in practice when, after failure of the first component, the failure rates for the remaining operative but not-working components either decrease to zero or become negligible.

Using the logic expressed in Equations 11.4 and 11.5, the failure rate and average outage duration of a general \( n \)-component series system may