NUMERICAL ANALYSIS OF SURFACE RELIEF GRATINGS

R. Orta, S. Bastonero, and R. Tascone

Electronics Department and CESPA (C.N.R.)
Polytechnic of Turin,
Corso Duca degli Abruzzi 24, 10129, Torino

1. INTRODUCTION

Diffraction gratings are components of great interest for their many applications, for instance in integrated optics, holography and spectroscopy. Among the methods introduced for the analysis of diffraction gratings one may recall the integral method, the coupled wave method, and the differential method. In this paper we will discuss a modal method for the analysis of lamellar surface relief diffraction gratings.

Due to the step-like form of the surface of such a grating (Fig. 1), the approach is to view the structure as a series of junctions between two periodic arrays of slab waveguides with the same period and different slab thickness and to compute the Generalized Scattering Matrix (GSM) of each junction. With the term GSM we indicate the scattering matrix relative to both propagating and cutoff slab modes. Then the characterization of the grating is obtained by combining the various GSMs.

In carrying out this program we will use a modal transmission line approach which is based on a reformulation of field theory in microwave network terms.

The structure of the Chapter is the following. Section 2 is devoted to a brief summary of the normal mode theory, with the essential purpose of defining the notation. Section 3 presents the solution of the basic scattering problem in the form of the GSM of the junction. Section 4 discusses how to compute the efficiency of the complete grating and some numerical results are presented.

2. REVIEW OF MODE THEORY FOR DIELECTRIC WAVEGUIDES

To establish the notation, let us review the basic facts of normal mode theory. Consider a dielectric waveguide with axis parallel to \( \hat{z} \). As well known, the transverse components of the electric and magnetic field are the independent state variables, whereas the longitudinal variables \( E_z, H_z \) are dependent and can be eliminated. By carrying out this
Fig. 1 Geometry of a lamellar surface relief diffraction grating

procedure, which requires the decomposition of all vectors in transverse (subscript $t$) and longitudinal components, the Maxwell equations become:

\[
\begin{align*}
- \frac{\partial}{\partial z} E_t &= j\omega \left[ \frac{\mu}{\omega^2} \nabla_t \frac{1}{\varepsilon} \nabla_t \right] \cdot (H_t \times \hat{z}) + M \times \hat{z} \\
- \frac{\partial}{\partial z} H_t &= j\omega \left[ \frac{\varepsilon}{\omega^2} \nabla_t \frac{1}{\mu} \nabla_t \right] \cdot (\vec{E}_t \times \hat{z}) + \vec{E}_t \times \hat{z}
\end{align*}
\]

(1)

where $I$ and $M$ are transverse electric and magnetic current densities. The temporal factor $\exp(j\omega t)$ is implied and suppressed. Dyadics are underlined twice and $\mathbb{1}$ denotes the identity dyadic in the transverse plane. Eq. (1), completed with the appropriate boundary conditions, are called the Marcuvitz - Schwinger equations and form the modern mathematical basis for the formulation of all types of waveguide problems.

The transverse electric and magnetic fields can be expressed as linear combinations of modes, i.e. source free solutions of Eq. (1):

\[
E_t(p, z) = \sum_i V_i(z) \mathbf{e}_i(p) \quad H_t(p, z) = \sum_i I_i(z) \mathbf{h}_i(p)
\]

(2)

In general, the normal modes of dielectric waveguides are hybrid and the mode vector eigenfunctions $\mathbf{e}_i, \mathbf{h}_i$ possess nonvanishing longitudinal components $e_{zi}, h_{zi}$. The expansion coefficients $V_i, I_i$ are the modal voltage and current and satisfy the transmission line equations

\[
\begin{align*}
- \frac{dV_i}{dz} &= jk_{zi} Z_i I_i + v_i \\
- \frac{dI_i}{dz} &= jk_{zi} Y_i V_i + i_i
\end{align*}
\]

(3)