EXPERIMENTS ON RYDBERG WAVE PACKETS

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1. INTRODUCTION

1.1 The concept of a wave packet

Under the influence of the development of lasers producing tunable ultra-short pulses, there has been an increased interest in the behavior of electronic wave packets during the last few years. An extensive overview of the field, with an emphasis on the theory, has been given in two theoretical papers¹,². The central theme of the present paper is the experimental observation of atomic electron wave packets.

Wave packets form a standard topic in almost any text book on quantum mechanics. The concept originates from Schrödinger³, who in 1926 calculated the behavior of a superposition of stationary wave functions for an electron in a harmonic potential well. He showed that the created wave packet oscillates with the same frequency as a classical particle, and that its centre-of-mass follows the classical trajectory. Although the concept of a wave packet has always been recognized as being of fundamental importance, experimental observation of 'resonances' in the frequency domain is usually much easier. Therefore, in general, more attention is paid to the properties of stationary states, i.e. the eigenfunctions Ψₙ and the eigenenergies Eₙ. Here n denotes a set of quantum numbers defining the state. Each quantum number is connected to a conserved physical quantity like energy, angular momentum, spin etc. The eigenfunctions and eigenenergies can be calculated by solving the time-independent Schrödinger equation, H Ψ = E Ψ. If the system is in a stationary state, the wave function has the form Ψₙ(r)·exp(-i Eₙ t). Unless indicated otherwise, atomic units are used throughout this article. As a consequence energy and frequency have the same numerical value: Eₙ = ωₙ. The probability distribution of the stationary state, Pₙ (r) = |Ψₙ(r)|², does not
change with time, and the time-dependent phase factor \( \exp(i \omega_n t) \) has no direct physical consequences. If the system is in a non-stationary state, \( \Psi \) is time dependent and has the form of a travelling wave packet. As a result, the probability distribution is time-dependent as well. Coherent superposition of at least two non-degenerate states is required to have not only a time-dependent wavefunction, but also a time-dependent probability distribution. A quantum beat is a superposition of two eigenstates that have a small energy splitting. The development of the phase difference between the states is therefore relatively slow and hence potentially observable.

The quantum numbers describing an electron wave function in three-dimensional space do have a classical analog. A striking difference between classical and quantum mechanics is, that the first describes the behavior of moving point objects, while the latter describes the behavior of extended waves. The concept of a wave packet forms a bridge between these two extremes: localization in space, together with motion, can be described in quantum mechanics by constructing a wave packet. However, a wave packet does not describe the motion of a single classical particle: it represents the time-dependent probability distribution for an ensemble of particles, and exhibits all wave-like properties like interference, dispersion, partial reflection etc.

Being a quantum phenomenon, a wave packet always obeys the Heisenberg uncertainty relation. Therefore time resolution can only be obtained at the cost of frequency resolution and vice versa (the same is true for resolution in position versus momentum). Within the Schrödinger picture of quantum mechanics, the uncertainty can also be understood as a result of the fact that one describes the behaviour of waves. The Fourier transformation applies just as well for probability amplitude waves as for classical waves, so that the Heisenberg uncertainty relation can be considered as a special case of the Fourier theorem, \( \Delta \omega \Delta t \geq \frac{1}{2} \).

1.2 Coherent excitation

As mentioned, there is a tendency to describe a wave packet as a coherent superposition of eigenstates, rather than as a time-dependent wave function. We will stay in line with this tradition. The stationary Rydberg states will serve as building blocks for wave packets. However, we would like to warn that such an expansion does not necessarily give the best insight into the physics of the phenomenon.

Upon irradiation with light, a quantum system can be excited to a single state if the bandwidth of the light is smaller than the energy separation between adjacent states. However, if the excitation takes place with a short light pulse, the coherent bandwidth can become larger than the energy separation between the excited states (see figure 1). In that case, not a single but several eigenstates are coherently excited:

\[
\Psi(r,t) = \sum_n a_n e^{-i\omega_n t} \Psi_n(r)
\]  

In eqn. 1.1 the wave packet \( \Psi(r,t) \) is written as a superposition of all the normalized eigenstates. The complex amplitudes \( a_n \) determine the populations \( |a_n|^2 \) of the various eigenstates. Population of several eigenstates instead of one means that on the one hand the energy of the excited wave packet is no longer well-defined, while on the other hand it has acquired a time-dependent structure: time resolution is gained at the cost of energy resolution (the Heisenberg uncertainty relation). Finally, coherent excitation implies that a definite phase relation be-